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Expectations and chaotic dynamics: Empirical evidence on exchange rates

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Abstract

The paper investigates the dynamic behavior of exchange rates expectations using four different currencies. The test follows developments advanced by Fernández-Rodríguez et al. [Fernández-Rodríguez, F., Sosvilla-Rivero, S., Andrada-Félix, J., 2005. Testing chaotic dynamics via Lyapunov exponents. Journal of Applied Econometrics, 20, 911–930.]. The evidence, however, does not favor the presence of chaotic dynamics in exchange rate expectations.

Keywords: Deterministic chaos; Exchange rates; Expectations; Lyapunov exponent

JEL classification: F31; F47

1. Introduction

The complex dynamic behavior of exchange rates has motivated a broad empirical literature that focused on the underlying structure of the data generating process and the predictability associated with different models. Complex non-linear dynamics can be associated in different theoretical settings with a simple deterministic process that induce a substantial sensitivity to the initial conditions of the dynamic system.

An important challenge is therefore to detect deterministic chaos in empirical applications. In that sense, the evidence appears to be mixed as exemplified by Frank and Stengos (1988), Pinder (1996), Bajo-Rubio et al. (1992) and Fernández-Rodríguez et al. (2005) in different contexts. The last two articles consider the evidence for exchange rates and show only partial support for the prevalence of deterministic chaos. A central tool for the referred investigation is the estimation of the largest Lyapunov exponent that has to be considered in terms of recently developed algorithms that are reliable for samples of moderate size.

The present paper intends to explore more primal aspects that might be generating a complex dynamics in exchange rates, namely the related expectations. The difference between testing for dynamics in exchange rates and in exchange rates expectations is relevant because many models allow for chaotic behavior in the former but not necessarily on the underlying expectations. For instance, Chappell (1997) presents an inflation model where it is shown that when expectations are adaptive the model can exhibit chaotic behavior for a range of plausible parameter values, but with rational expectations chaotic behavior is not possible. Kyrtsou et al. (2004) shows that future prices movements can be random, suggesting noisy chaotic behavior. Specifically regarding exchange rates models, De Grauwe and Dewachter (1993) and De Grauwe et al. (1995) develop a model that can generate a generate a chaotic nominal exchange rate through the interaction of chartists and fundamentalists in forward exchange rates markets. Da Silva (2001) generalizes the results, and chaotic solutions are still shown to be possible for sensible parameter values. Federici and Gandolfo (2002) further advance the literature by allowing continuous time in the models. Fernández-Rodríguez et al. (2005) finds evidence of chaos in exchange rates. However, these models and applications do not allow for chaotic behavior in expectations.

Resende (2000) had identified non-linear dynamics in the context of exchange rates expectations, but the empirical

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literature on expectations, as indicated by Dutt and Ghosh (1997), has concentrated on the evidence of rationality. Clearly a gap remains in terms of a deeper understanding of the nature of the non-linear dynamics. The link between expectations and the realized exchange rate is not fully explored and is thus a relevant research question. We undertake a step further to verify whether the evidence on exchange rates expectations is consistent with deterministic chaos. For that purpose we obtain Lyapunov exponents for different currencies' expectations upon weekly data within the 1984–88 period. The refining procedure advanced by Fernández-Rodríguez et al. (2005) that considers the stability of the average of the largest Lyapunov exponents across different sub-samples is implemented.

The paper is organized as follows. Section 2 briefly discusses the basic concepts and testing strategy to detect deterministic chaos. Section 3 discusses the data source and presents the empirical results. Section 4 brings some final comments.

2. Chaotic dynamics: basic conceptual aspects

Lyapunov exponents constitute a central concept for determining the sensitivity to changes on the initial conditions of a dynamic system. The exponents provide the averaged rate of exponential divergence (or convergence) from perturbed initial conditions. The intuition can be made clear in the case for a one dimensional discrete dynamical system given by $x_{k+1}=f(x_k)$ with initial condition x_0. The Lyapunov exponent is then:

$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \ln|f'(x_0)| \tag{1}$$

The derivative indicates the extent of the change in the transition from x_k to x_{k+1}, whereas the logarithmic transformation provides a change in scale. One therefore has an average of the log of the derivatives over n iterations. The Lyapunov exponent indicates the average rate of divergence between two neighbouring starting points following some perturbation. A positive Lyapunov exponent would indicate chaotic dynamics.

Different algorithms were developed for computing Lyapunov exponents with small samples [e.g. Wolf et al. (1985), Gençay and Dechert (1992), Rosenstein et al. (1993), and Mathiesen and Cvitanović (2005)]. That class of procedures assumes that the initial rate of divergence grows at exponential rate given by the maximum Lyapunov exponent ($\lambda_{\max}$) in the aforementioned reconstructed state space of a time series. The key parameters for the computation are (Fernández-Rodríguez et al., 2005): the embedding dimension (d), the lag or reconstruction delay, the mean period; and the number of discrete time steps (i) allowed for divergence between nearest neighbours in the phase space.

Fernández-Rodríguez et al. (2005) further advance by providing a statistical foundation for the estimation and analysis of Lyapunov exponents. The problem is that the largest Lyapunov exponent on its own is not able to distinguish between a chaotic, non-linear deterministic process and a random process. The authors show the importance of evaluating the stability of the largest Lyapunov exponents across different sample sizes and propose a test using bootstrap aggregation, or bagging, a procedure that averages the estimations over a collection of bootstrap samples, thereby reducing the variance (Table 1).

Dividing the sample in terms of sub-samples that contains the precedent, one obtains moving blocks. A crucial result is that the largest Lyapunov exponents stabilizes or decreases as the sample size increases in a deterministic process whereas it would increase with sample size in the case of a stochastic process,

$$\lambda_{\max}(T) = \alpha_0 + \alpha_1 T + \varepsilon_T \tag{2}$$

so that the estimated parameter $\alpha\hat{}$ can be used to test if the largest Lyapunov exponent does not increase with sample size, which is a feature of chaotic processes. The null hypothesis would be given by H_0:$\alpha_1 \leq 0$ (deterministic process) whereas the alternative hypothesis H_1:$\alpha_1 > 0$ would be associated with a stochastic process. Therefore, the regression based on averages of the maximum Lyapunov exponents across the different bootstrapped samples provide the elements for conducting the relevant test.

3. Empirical application

3.1. Data

The basic data source was the *New York Money Market Survey* (NYMMS) based on phone surveys conducted on a weekly basis since 1984. Approximately 30 traders declared their expectations with respect to future exchange rates for British pound (BP), the Deutschmark (DM), the Japanese yen (JY) and Swiss franc (SF) at a horizon of one week ahead. The series have data from 10/24/1984 to 01/08/1988, comprising 168 observations. Despite the sample period, the data set has particular interest by providing information on expectations, which is unusual. Resende (2000) had previously used the same data set for investigating the presence of non-linear dynamics. We build on that paper that had removed linear structure on the data by identifying the relevant ARIMA model and considering the associated residuals in the analysis. We follow that preliminary procedure by considering the residuals

Table 1
Tests on the stability of the largest Lyapunov exponents

		DIM2	DIM3	DIM4	DIM5	DIM6
JY	Coefficient	0.0010	0.0008	0.0012	0.0005	−0.0001
	p-value	0.0002	−0.0003	0.0005	0.0002	0.8865
SF	Coefficient	0.0007	0.0008	0.0012	0.0005	0.0009
	p-value	0.0069	0.0003	0.0006	0.0012	0.0003
DM	Coefficient	0.0033	0.0017	0.0015	0.0017	0.0010
	p-value	0.0009	0.0000	0.0009	0.0007	0.0007
BP	Coefficient	0.0000	0.0014	0.0007	0.0009	0.0002
	p-value	0.0001	0.0006	0.0010	0.0626	0.0016

Note: The different dimensions indicated in the table stand for the embedding dimensions.

of the following ARIMA models: BP (1,1,2); DM (1.1.1); JY (4,1.2); SF (1,1,1) identified by that paper.

3.2. Empirical results

The necessary parameters were selected in the same vein as Fernández-Rodríguez et al. (2005), with one difference. We find that estimation was not relatively invariant to the choice of embedding dimension. In our case, results differed depending on the choice value of embedding dimension.

For the other parameters the lag, or reconstruction delay, has been fixed to 1 in all cases. The subsamples were defined as: $T_1=88$, $T_2=93$, …,$T_{17}=168$. As in Fernández-Rodríguez et al. (2005, p. 920) estimation of the dominant Lyapunov exponent for sample size T_i was $<\lambda_{\max}(T_i)>$, the mean of the distributions of the largest Lyapunov exponents computed from the sample sizes T_i, which corresponds to the bootstrap aggregation or bagging of 100 bootstrap samples of the largest Lyapunov exponents. Results are as follows.

The results strongly indicate that there is no presence of chaotic behavior in exchange rate expectations, since only for one dimension in BP and one in DM there is indication of chaos. It would be expected that if chaos was present it would show across the board invariant to the dimension, like in Fernández-Rodríguez et al. (2005).

4. Final comments

The paper aimed at investigating non-linearities in the behavior of exchange rates for different currencies. In particular, it addresses a gap in the literature in what concerns the prevalence of complex patterns in exchange rate expectations. For that purpose, we have relied on an unusual data set and pursued the assessment of the presence of deterministic chaos. The evidence, however, does not support the existence of a chaotic dynamics in exchange rate expectations. It is important, however, to emphasize that this evidence does not rule out the possibility of complex dynamics in exchange rates, since models that allow for complex dynamics in exchange rate do not necessarily rely on agents' expectations to create such dynamics (e.g. De Grauwe and Dewachter, 1993; Federici and Gandolfo, 2002).

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