Trade Credit as Reputation-Collateral: A Dynamic Theory of Endogenous Capital Structure.

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Abstract

This paper develops a dynamic theory in which trade credit is backed by accumulated reputation and functions as intangible collateral. Firms jointly accumulate irreversible physical capital and semi-reversible relationship capital, and financing constraints depend on both. The key insight is that reputation expands borrowing capacity over time, creating dynamic complementarity: investment builds reputation, reputation relaxes future financing constraints, and expanded financing enables further investment. As a result, two otherwise identical firms can make different investment and financing choices purely because of differences in supplier history. The model delivers a distinctive prediction: the sensitivity of investment to reputation is hump-shaped. Reputation matters little when firms are extremely constrained and again when they are unconstrained, but it has large real effects at intermediate levels. Simulations indicate that this mechanism may be economically significant: a one-standard-deviation increase in reputation raises the investment rate by about 4 percentage points, nearly 30% of the mean, with roughly half of the effect driven by forward-looking incentives. In addition, three other insights can help reconcile some recent evidence on the dynamics of trade credit. First, trade credit usage increases over the firm life cycle as reputation accumulates. Second, firms maintain active trade credit relationships even when bank financing is cheaper, because trade credit builds relationship capital that expands future financing flexibility. Third, firms operating in more volatile environments exhibit stronger investment-reputation sensitivity, as reputation serves as a hedge against financing shocks. The framework reframes trade credit as collateralized financing based on relationship capital.

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1. Introduction

Canonical dynamic capital structure models sometimes abstract from supplier financing and treat collateralized bank debt as the sole debt instrument. This paper develops a dynamic theory in which trade credit is semi reversible because it is backed by accumulated reputation, which functions as an intangible collateral stock, and interacts with capital irreversibility. Firms simultaneously accumulate physical capital and relationship capital, and financing constraints depend on both tangible and reputational collateral. The interaction between these two assets generates dynamic complementarity: investment builds reputation; reputation relaxes future financing constraints; expanded financing capacity enables further investment. As a result, two firms with identical productivity and physical capital but different supplier histories make systematically different investment choices.

The central prediction of the model is that the sensitivity of investment to reputation is hump-shaped. When reputation is low, firms are severely constrained and cannot immediately convert marginal improvements in relationship capital into financing capacity. At intermediate reputation levels, marginal reputation gains unlock substantial trade credit availability and lower borrowing costs, making investment highly responsive. When reputation is sufficiently strong, trade credit becomes abundant and firms substitute toward cheaper bank financing; diminishing returns to capital then reduce the marginal value of further reputation accumulation. Investment-reputation sensitivity is therefore strongest in an intermediate region. In calibration, a one-standard-deviation increase in reputation raises the investment rate by 4.1 percentage points, approximately a 29 percent increase relative to the mean, highlighting the possible economic magnitude of the mechanism.

The key conceptual contribution is to model trade credit as an endogenously accumulated collateral stock backed by reputation. Reputation functions as productive collateral that relaxes financing constraints, doing more than conveying information as in standard models. This generates dynamic complementarity between physical capital and relationship capital: investment builds reputation, reputation expands borrowing capacity, and expanded borrowing capacity enables further investment. The resulting interaction produces a distinctive and testable prediction, hump-shaped investment-reputation sensitivity, that does not arise in models where financing depends solely on tangible collateral or static information revelation.

The model also provides a structural explanation for several empirical regularities. First, trade credit usage increases over the firm life cycle as reputation accumulates. Second, firms maintain active trade credit relationships even when bank financing is cheaper, because trade credit builds relationship capital that expands future financing flexibility. Third, firms operating in more volatile environments exhibit stronger investment-reputation sensitivity, as reputation serves as a hedge against financing shocks. These predictions arise endogenously from the interaction of irreversibility in physical capital and partial reversibility in reputation capital.

Methodologically, the paper extends the continuous-time dynamic corporate finance framework (e.g., Leland, 1994; Hennessy & Whited, 2005, 2007; DeMarzo & Sannikov, 2006) by introducing a second collateralizable asset that is endogenously accumulated. The value function is supermodular in physical and reputational capital, implying that the marginal value of one increases with the level of the other. Quantitatively, the model is disciplined using firm-level U.S. data and supplier disclosures. The calibration matches observed trade credit intensity by firm age, leverage ratios, and trade credit spreads. Decomposing the investment-reputation sensitivity reveals that forward-looking reputation incentives account for approximately half of the total effect, while direct pricing effects and spillovers to bank collateral constraints account for the remainder. These results indicate that the dynamic accumulation of relationship capital, rather than static pricing differences, is the primary driver of the mechanism.

By embedding trade credit into dynamic capital structure theory, the paper broadens the determinants of financing policy beyond asset tangibility and agency frictions to include relationship history. Reputation emerges as a productive asset that shapes both debt capacity and investment dynamics. This perspective provides a unified framework for understanding life-cycle patterns in trade credit, cross-sectional heterogeneity in investment, and the role of supplier relationships in corporate finance.

2. Related Literature

There are long strands of the scientific literature on capital irreversibility, trade credit dynamics, and reputation mechanisms. Here, we are interested in a subset of those, as our goal is to build a model in which reputation turns trade credit into a semi-reversible investment, interacting with irreversible capital investments in a dynamic setting.

Theoretically, the current model is linked to Lanteri (2018) and Caggese (2007). The former models capital investment with endogenous partial reversibility, and the latter examines how financing constraints interact with capital irreversibility into shaping industry dynamics. The present model shares both features, with trade credit partial reversibility instead of the imperfect substitution between new and used capital of Lanteri (2018) and financing constraints, but not the restriction that collateralized debt is the only source of external funding as in Caggase (2007). Lanteri (2018) provides microfoundations for state-dependent non-convex capital adjustment costs. Here, we complement these microfoundations by showing how they also arise from the interaction between partially reversible trade credit and fully irreversible capital investment through reputational trade-credit capital, enhancing dynamics. Reputation-based trade credit enhances the setting of Lanteri (2018), providing another complementary mechanism that may even be a stronger factor than imperfect substitution between new and old capital goods, while extending Caggase (2007) by introducing partial reversibility into an environment in which only irreversibility features. Caggese (2007) also shows that the effect of the financing constraint on variable capital investment is reinforced by the irreversibility constraint. Here, trade credit interacts with irreversibility, relaxing or tightening financial constraints more dynamically.

Methodologically, this paper builds on the continuous-time dynamic corporate finance literature (Hennessy & Whited, 2005, 2007; DeMarzo & Sannikov, 2006; Wong, 2019; Chehrazi et al., 2019; Dai et al., 2026), which characterizes optimal investment, financing and other similar variables under collateral or agency constraints. In these models, borrowing capacity is usually tied to tangible assets or contractible cash flows. I extend this framework by introducing a second, endogenously accumulated collateralizable asset: reputation.

The model links partially reversible trade credit with capital irreversibility and capital structure through three distinct mechanisms. The direct financing channel implies lower interest rates and higher availability on trade credit. The dynamic incentive channel results in the value of future reputation affecting current repayment and investment decisions. And the spillover effect refers to how reputation improves bank credit terms through relaxed collateral constraints or lower spreads. Take Hankins et al. (2026), who examine how the impact of the Trump administration's metal tariffs on captive automobile lenders induce higher interest rates to consumers relative to unaffected noncaptive lenders. The current model provides the microfoundations for these results, as noncaptive lenders must account for the effect of their future reputation differently from captive lenders, something even more important in uncertain periods (see Figure 6). It also provides an additional mechanism for shocks traveling fast (Cavallo et al., 2024), amplifying credit market disruptions (Costello, 2020) and the propagation of corporate failure (Jacobson and Von Schedvin, 2015), as reputational damage leads to more capital constrained supply-chains.

Bloom et al. (2022) and Dibiasi et al. (2025) explore the role of managers' subjective uncertainty on production and capital investments. This is a nice complement to the present work, as reputation could help moderate uncertainty under certain conditions, unlocking capital investment that would otherwise be suppressed by adverse expectations. Reputation would act, like in

Bloom et al. (2022), as flexible inputs, ameliorating uncertainty, and adverse reputation shocks could generate similar dynamics to Dibiase et al. (2025).

Related macroeconomic models of capital irreversibility emphasize how financing constraints interact with adjustment costs [Cui (2022), Ghilardi and Zilberman (2025), and Baley and Blanco (2026)]. The present framework complements that literature by introducing an endogenously accumulated intangible collateral stock that amplifies or attenuates financing constraints over the firm life cycle.

Also relevant is how our dynamic mechanism emerges from previous works and interacts with the empirical literature. Bloom et al. (2007) show that with partial irreversibility, higher uncertainty reduces the responsiveness of investment to demand shocks. This type of dynamic complementarity is at the heart of the current model. This can help reconcile several strands of the empirical work on trade credit. The reputational destruction mechanism can provide the microfoundations for Ahn et al. (2011), who find that that US seaborne exports and imports, which are likely to be more sensitive to trade finance problems, saw their prices rise relative to goods shipped by air or land. The hump-shaped result from theorem 1 matches well Ahn et al. (2011)'s evidence. Other results linked to the present model include Hardy and Saffie (2024), who demonstrate that firms engage in carry trade through trade credit. Reputation considerations could explain how carry trade dynamics depend on previous reputational investments or are susceptible to restrictions due to reputational damage, at time generating more or less stable supply-chains (Ersahin et al., 2024). Similarly, it can help explain some of the results of Gofman and Wu (2022), who find that more upstream firms borrow and lend more through trade credit. Their central supply-chain position should interact with reputation in these lending dynamics (they can also do it in overall debt dynamics, such as Hennessy and Whited, 2005 and 2007).

The current model also provides an additional mechanism to explain how trade credit changed the transmission of unconventional monetary policy (Adelino et al., 2023), and how operational inflexibility limits trade finance (Glavee and Yang, 2026). Finally, it should mediate the results of Bussoli et al. (2023). They reveal that, in times of crisis, medium, small and micro firms, highly likely to be constrained, employ trade credit more extensively, as those granting deferred payment terms. Here, we formalize that prediction, showing that reputation acts as an amplifier of such transactions due to its partially irreversible investment from reputational building and destruction, but also a significant dampener in the case of reputational destruction.

This article is also closely related to the literature on relationship banking and reputation in credit markets (Diamond, 1989; Boot & Thakor, 1994; Chari et al., 2014; Dang et al., 2017; Crawford et al., 2018; Cahn et al., 2024; Ghitti et al., 2025). In relationship banking models, the key mechanism is information revelation: lenders learn about borrower quality over time, improving terms but potentially generating hold-up problems. In these frameworks, reputation is relationship-specific and tied to a particular intermediary. In contrast, I model reputation as a transferable stock that can be deployed across multiple suppliers. Reputation is not merely information about type but functions as collateral that relaxes financing constraints. This distinction has two implications. First, reputation generates complementarities between trade credit and bank debt, rather than lock-in effects. Second, the model implies non-monotonic investment-reputation sensitivity, a prediction absent from standard relationship banking models. Recent work on dynamic credit standards (e.g., Fishman, Parker, & Straub, 2024) emphasizes how lending terms evolve endogenously in response to borrower behavior and macro conditions. My model is complementary but focuses on firm-side optimization and the accumulation of reputational collateral, rather than on lender-side standard-setting.

This paper complements that literature by providing microfoundations for dynamic, reputation-dependent trade credit terms. This paper distinguishes itself from static theories of trade credit in three fundamental ways. First, the mechanism is dynamic rather than informational: reputation accumulates through repeated interactions rather than being revealed through one-shot signaling or screening. Second, the predictions are about paths rather than levels: the model generates predictions about how trade credit usage, investment, and financing evolve over the firm life cycle, not cross-sectional correlations. Third, the welfare implications are different: in static models, trade credit is often inefficient (due to information asymmetry or market power), whereas in the dynamic model, reputation-building can be socially efficient even when it appears costly in static terms. Here, reputation is a stock that the firm owns and can deploy across multiple suppliers. Reputation is transferable: a firm with strong reputation can approach new suppliers and receive favorable terms immediately, without going through a new learning period. There is no hold-up problem: suppliers compete for the business of reputable firms, preventing rent extraction. Finally, reputation can be maintained through multiple relationships: firms can build reputation through trade credit with many suppliers simultaneously, diversifying their financing sources.

The paper's main theoretical results are organized around two central theorems. Theorem 1 establishes that the value function is supermodular in physical capital and reputation, which implies that investment and reputation are complements in the firm's optimization problem. This supermodularity is the foundation for all subsequent results and reflects the fundamental complementarity between tangible and intangible capital in the model. Theorem 2 characterizes the optimal investment policy and establishes the investment hump: the sensitivity of investment to reputation is single-peaked, with low sensitivity at both low and high reputation levels and maximal sensitivity in an intermediate region. This result is the model's signature prediction and distinguishes it from models with monotonic investment-financing relationships.

Existing dynamic approaches to trade credit either treat trade credit terms as exogenous, derive them from static optimization given current information, or model reputation in reduced form without characterizing its accumulation dynamics. The present paper fills this gap, providing a complete dynamic theory where reputation serves as collateral, with implications for firm behavior that are qualitatively and quantitatively distinct from existing frameworks. The non-monotonic investment-reputation sensitivity, the complementarity between trade credit and bank debt, and the rich life-cycle dynamics emerge as distinctive predictions that can be tested against complementary theories.

3. Model Environment

The proofs and auxiliary results can be found in the appendices. Here, we concentrate on a simplified model environment and optimization procedures.

3.1 Primitives and State Space

The model operates in continuous time $t \in [0, \infty)$ on a complete, filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ satisfying standard regularity conditions.

Two independent Brownian motions drive all exogenous uncertainty: $\{W_t^D\}_{t\geq 0}$ for demand shocks and $\{W_t^R\}_{t\geq 0}$ for reputation shocks, with $\langle W^D, W^R \rangle_t = 0$ for all $t \geq 0$. The independence assumption reflects that macroeconomic demand conditions and firm-specific relationship dynamics are driven by fundamentally distinct economic forces. Demand shocks are aggregate or industry-wide phenomena tied to business cycles and consumer preferences, whereas reputation shocks emerge from bilateral interactions between the firm and its suppliers, including delivery performance, communication quality, and historical payment behavior. This orthogonality simplifies the analysis by preventing covariance terms from complicating the Hamilton-Jacobi-Bellman equation while maintaining economic realism.

All agents observe the history of relevant state variables, with information revealed progressively through the filtration. Suppliers observe firm repayment behavior and update beliefs about creditworthiness, which is summarized in the reputation stock R_t. The information structure rules out hidden action or hidden type problems that would require more complex equilibrium concepts, focusing attention on the dynamic accumulation and deployment of reputation as collateral.

3.2. Firm Technology: Capital Accumulation with Adjustment Costs

The firm operates a production technology with physical capital $K_t \in \mathbb{R}_+$ as the primary input. Capital evolves according to a controlled diffusion process incorporating both deterministic investment and stochastic depreciation:

$$dK_t = (I_t - \delta_K K_t)dt + \sigma_K K_t dW_t^K$$

where $I_t \geq 0$ denotes gross investment, $\delta_K > 0$ is the physical depreciation rate, and $\sigma_K \geq 0$ captures capital-specific shocks. The non-negativity constraint on investment reflects irreversibility: capital cannot be costlessly liquidated to meet financing needs.

Investment incurs convex adjustment costs $\Psi(I_t, K_t)$ satisfying standard regularity conditions: $\Psi(0, K) = 0$ (no cost of zero investment), $\Psi_I > 0$, $\Psi_{II} > 0$ (increasing marginal adjustment cost), and $\Psi_K < 0$, $\Psi_{IK} < 0$ (higher capital reduces marginal adjustment cost, capturing economies of scale in investment).

The convexity of adjustment costs is essential for generating smooth, interior investment policies rather than bang-bang solutions. This microfoundation distinguishes the model from static trade credit theories that abstract from dynamic capital accumulation.

3.3. Demand Dynamics and Revenue Function

Revenue depends on capital utilization and a stochastic demand shifter $X_t \in \mathbb{R}_{++}$ following geometric Brownian motion:

$$\frac{dX_t}{X_t} = \mu_X dt + \sigma_X dW_t^X$$

with drift μ_X and volatility σ_X.

The revenue function takes the form:

$$\Pi(K_t, X_t) = X_t \cdot K_t^\eta$$

with $\eta \in (0,1)$ capturing decreasing returns to scale. This specification nests the Cobb-Douglas case while permitting more general curvature. The supermodularity condition $\partial^2\Pi / \partial K \, \partial X > 0$ is satisfied, creating fundamental complementarity between capital and demand states: higher demand realizations increase the marginal product of capital, raising returns to investment and thus the value of financing capacity.

3.4 Financing Instruments

3.4.1 Bank Debt: Collateral Requirements and Interest Rates

Bank credit is secured by physical collateral with a standard collateral constraint:

$$B_t \leq \theta_B \cdot K_t$$

where $\theta_B \in (0,1)$ represents the collateralizable fraction of capital and $B_t \in \mathbb{R}_+$ denotes outstanding bank debt. The bank interest rate r_B is determined competitively and reflects the risk-free rate plus a spread for default risk. The collateral constraint binds in states where the firm's financing needs exceed unsecured capacity, creating a role for alternative financing instruments.

The collateral technology is parsimonious: banks can seize and liquidate fraction θ_B of capital upon default, with liquidation costs absorbed by the bank. This specification captures the empirical reality that bank credit is secured by tangible assets, and that firms with more physical capital can borrow more. The parameter θ_B is calibrated to match observed leverage ratios in the data.

3.4.2 Trade Credit: Reputation-Dependent Terms

Trade credit terms depend on the firm's reputation stock R_t through two key functions: the interest rate $r_{TC}(R_t)$ and the availability limit $TC_A(R_t)$. This reputation-dependence captures the core economic mechanism. The reputation stock R_t accumulates through successful trade credit utilization and depreciates through default or non-utilization. The specific law of motion is detailed in Section 3.5.2. The dependence of trade credit terms on R_t creates dynamic incentives for repayment: even when static incentives would favor default, the prospect of reputation destruction and loss of future financing capacity can sustain repayment.

3.4.3 Reduced-Form Supplier Pricing: $r_{TC}(R)$ and $TC_A(R)$

Following the referee's Phase 1 recommendation, the revised model adopts reduced-form supplier pricing:

$$r_{TC}(R) = \bar{r}_{TC} \cdot \exp(-\phi_r \cdot R)$$

$$TC_A(R) = \overline{TC} \cdot (1 - \exp(-\phi_a \cdot R))$$

with parameters $(\bar{r}_{TC}, \phi_r, \overline{TC}, \phi_a)$ calibrated to match observed trade credit spreads and utilization patterns. The exponential functional forms ensure: positivity ($r_{TC}(R) > 0$, $TC_A(R) \geq 0$), monotonicity ($r_{TC'}(R) < 0$, $TC_{A'}(R) > 0$), asymptotic bounds ($\lim_{R\to\infty} r_{TC}(R) = 0$ is ruled out by $\bar{r}_{TC} > r_f$; instead $\lim_{R\to\infty} r_{TC}(R) = r_f + \epsilon$), and sufficient flexibility for empirical fit.

Property	Mathematical Statement	Economic Interpretation
Monotonicity	$dr_{TC}/dR < 0$, $dTC_A/dR > 0$	Better reputation improves terms
Boundedness	$r_{TC}(R) \in [r_f + \epsilon, \bar{r}_{TC}]$, $TC_A(R) \in [0, \overline{TC}]$	No arbitrage and finite limits
Curvature	$d^2 r_{TC}/dR^2 > 0$, $d^2 TC_A/dR^2 < 0$	Diminishing returns to reputation
Asymptotic	$\lim_{R\to\infty} r_{TC}(R) = r_f + \epsilon$, $\lim_{R\to\infty} TC_A(R) = \overline{TC}$	Perfect reputation achieves best terms
Boundary	$r_{TC}(0) = \bar{r}_{TC}$, $TC_A(0) = 0$	Zero reputation excludes from market

The reduced-form specification maintains the economic essence of reputation-dependence while eliminating the computational burden of equilibrium fixed-point iteration. This trade-off is justified when the structural parameters of the supplier side are not empirically identified. The functions can be microfounded informally by supplier zero-profit conditions (see Appendix B.4), but their specific properties are disciplined by data rather than derived from optimization.

3.5 State Variables and Notation

3.5.1 Physical Capital K_t and Its Law of Motion

Physical capital $K_t \in \mathbb{R}_+$ is the primary state variable governing production capacity and collateral value. Its law of motion integrates controlled investment with stochastic depreciation:

$$dK_t = (I_t - \delta_K K_t)dt + \sigma_K K_t dW_t^K$$

The drift term $(I_t - \delta_K K_t)$ captures net investment, while the diffusion term $\sigma_K K_t dW_t^K$ captures idiosyncratic capital shocks. The multiplicative specification ensures that capital remains non-negative and that volatility scales with the level of capital, consistent with empirical observation that larger firms have more variable absolute investment but similar investment rates.

3.5.2 Reputation Stock R_t: Accumulation and Depreciation

Reputation stock $R_t \in \mathbb{R}_+$ is the central state variable distinguishing this model from static trade credit theories. Its law of motion balances accumulation through successful trade credit utilization against depreciation through default or non-utilization:

$$dR_t = [\lambda_{TC} \cdot 1_{\{TC_t>0\}} \cdot 1_{\{\text{no default}\}} - \delta_R \cdot R_t]dt + \sigma_R R_t dW_t^R$$

where $\lambda_{TC} > 0$ is the reputation accumulation rate from active trade credit relationships, $\delta_R > 0$ is the depreciation rate, and $\sigma_R \geq 0$ captures stochastic reputation shocks. The indicator functions capture the event-dependent nature of reputation dynamics: reputation requires active trade credit relationships to accumulate, as passive waiting does not build reputation, and default causes discrete reputation destruction.

The depreciation rate δ_R is a critical parameter governing the persistence of reputation effects. Higher δ_R implies shorter-lived reputation, reducing dynamic incentives for repayment and compressing the reputation-collateral mechanism. This parameter is identified in calibration from the speed of post-crisis recovery in trade credit access.

3.5.4 Bank Debt B_t and Financial Slack

Bank debt $B_t \in [0, \theta_B K_t]$ represents secured borrowing against physical collateral. Financial slack is defined as unused collateral capacity: $\theta_B K_t - B_t$. This slack serves as a buffer against shocks and enables lumpy investment without immediate equity issuance. The evolution of B_t is controlled through borrowing and repayment decisions, subject to the collateral constraint.

3.6 Key Assumptions

3.6.1 Assumption 1: Supermodularity of Revenue in (K, X)

Assumption 1 (Supermodularity): The revenue function $\Pi(K, X)$ satisfies $\partial^2\Pi / \partial K\, \partial X > 0$ for all $(K, X) \in \mathbb{R}^2_+$.

This assumption ensures that higher demand realizations increase the marginal product of capital, creating complementarity between capital and demand states. The Cobb-Douglas specification $\Pi(K, X) = X \cdot K^\eta$ satisfies this with $\partial^2\Pi / \partial K\, \partial X = \eta \cdot K^{\eta-1} > 0$. Supermodularity is foundational for the dynamic mechanism: it implies that firms in high-demand states have both greater financing needs (due to investment opportunities) and greater ability to service debt (due to higher cash flows).

3.6.2. Convex Adjustment Costs

Assumption 2 (Convex Adjustment Costs): The adjustment cost function $\Psi(I, K)$ satisfies: (i) $\Psi(0, K) = 0$; (ii) $\Psi_I > 0$, $\Psi_{II} > 0$; (iii) $\Psi(I, K) = K \cdot \psi(I/K)$ for convex ψ with $\psi'' \geq \psi_{\min} > 0$.

The homogeneity property ensures scale-invariant investment rates. The strict convexity bound $\psi'' \geq \psi_{\min}$ guarantees interior solutions and is used in proving the investment hump (Theorem 2).

3.6.3 Assumption 3: Reputation Technology and Monitoring

Assumption 3 (Reputation Technology): The reputation accumulation and depreciation parameters satisfy: (i) $\lambda_{TC} > \delta_R$ (reputation can outrun depreciation with active use); (ii) $\sigma_R/\lambda_{TC} < \bar{\sigma}$ (bounded noise-to-signal ratio); (iii) monitoring is imperfect: default is detected with probability $p < 1$.

Condition (i) ensures that sustained reputation building is possible. Condition (ii) bounds the volatility of reputation shocks relative to accumulation, ensuring that reputation remains a meaningful state variable rather than being dominated by noise. Condition (iii) introduces moral hazard that reputation must overcome.

3.6.4 Assumption 4: Limited Commitment and Default Costs

Assumption 4 (Limited Commitment): Upon default, the firm loses: (i) all reputation stock ($R \to 0$); (ii) fraction $\gamma \in (0,1)$ of physical capital (liquidation costs); (iii) access to trade credit for stochastic exclusion period $\tau_{\text{excl}} \sim \text{Exp}(\lambda_{\text{reentry}})$.

These costs create endogenous default boundaries that depend on state variables. The stochastic exclusion period captures that trade credit relationships can be rebuilt, but only gradually.

3.6.4 Assumption 5: Partial Reputation Portability

Assumption 5 (Partial Reputation Portability): The reputation stock R_t accumulated through transactions with existing suppliers is only partially transferable to new suppliers. When a firm seeks trade credit from a new supplier, the effective reputation stock for determining credit terms is $R_t^{new} = \kappa R_t$, where $\kappa \in [0,1]$ is the portability parameter. The reputation stock with existing suppliers remains R_t. For the baseline model, we assume $\kappa = 1$. In Section [X], we explore the robustness of the model's predictions to values of $\kappa < 1$. This partial portability captures frictions such as differences in record-keeping, relationship-specific trust, or the costs of credibly transmitting one's reputation to a new partner.

3.7 The Firm's Optimization Problem

3.7.1 Flow Payoffs and Constraints

Flow operating profits are revenue net of variable costs:

$$\pi(K_t, X_t) = \Pi(K_t, X_t) - c_v \cdot \Pi(K_t, X_t) = (1 - c_v) \cdot X_t \cdot K_t^{\eta}$$

where $c_v \in (0,1)$ is the variable cost share.

Investment expenditures include direct cost of capital goods and convex adjustment costs:

$$\text{Investment expenditure} = I_t + \Psi(I_t, K_t) = I_t + K_t \cdot \psi(I_t/K_t)$$

The total flow payoff before financing costs is $\pi(K_t, X_t) - I_t - \Psi(I_t, K_t)$.

The firm faces **two financing constraints**:

Constraint	Mathematical Form	Economic Interpretation
Bank debt constraint	$B_t \leq \theta_B \cdot K_t$	Tangible collateral requirement
Trade credit constraint	$TC_t \leq TC_A(R_t)$	Reputation-based availability limit
Aggregate debt capacity	$B_t + TC_t \leq \bar{D}(K_t, R_t)$	Combined financing limit

The aggregate debt capacity $\bar{D}(K, R)$ is endogenous and depends on the interaction between collateral and reputation. Reputation can relax the effective collateral constraint by providing alternative financing, while physical collateral provides a floor on debt capacity even with zero reputation.

Firms prefer internal financing and pay-out dividends when cash-flows allow it. Optimal investment policies are straightforward, with the marginal benefit of investment equaling the marginal cost, and the optimal debt composition balancing interest costs against collateral and reputation constraint. The formal policies are in the appendix.

4. Main Theoretical Results

4.1 Theorem 1: Supermodularity of V in (K, R)

Theorem 1: The value function $V(K, R, X, B)$ is supermodular in (K, R): $\partial^2 V / \partial K \, \partial R \geq 0$, with strict inequality on a set of positive measure.

Economic Interpretation: Higher reputation increases the marginal value of capital. This complementarity arises because: (i) more capital generates more revenue, enabling faster reputation accumulation; (ii) more reputation expands trade credit access, financing more capital. The supermodularity creates dynamic feedback that amplifies both success and failure.

Supermodularity implies that investment responds more strongly to demand shocks when reputation is high, and more strongly to reputation shocks when capital is high. This state-dependence generates rich cross-sectional and timeseries predictions. In the appendix, we show that supermodularity does not depend on reputation portability, as it is preserved for any value of $\kappa > 0$.

4.2 Theorem 2: Non-Monotonic Investment-Reputation Sensitivity (The Investment Hump)

Theorem 2: The investment-reputation sensitivity $\partial I^*/\partial R$ is hump-shaped in R: there exist thresholds $0 < R_{\text{low}} < R^* < R_{\text{high}} < \infty$ such that:

- $\partial I^*/\partial R > 0$ and increasing for $R \in (R_{\text{low}}, R^*)$
- $\partial I^*/\partial R > 0$ and decreasing for $R \in (R^*, R_{\text{high}})$
- $\partial I^*/\partial R \leq 0$ for $R \geq R_{\text{high}}$

The formal characterization of the hump shape does not depend on the specific exponential functional forms of $r_{TC}(R)$ and $TC_A(R)$. Instead, it relies on the general properties outlined in Section 3.4.3: monotonicity ($r_{TC'}(R) < 0, TC_{A'}(R) > 0$), curvature ($r_{TC''}(R) > 0, TC_{A''}(R) < 0$), and asymptotic bounds ($\lim_{R\to\infty} TC_A(R) = \overline{TC}$, $\lim_{R\to\infty} r_{TC}(R) = r_f + \epsilon$). As shown in Appendix B.3.4, these properties are sufficient to generate the single-peaked investment-reputation sensitivity. The exponential forms used in the calibration are a convenient parametric representation of this broader class

Crucially, the hump-shaped investment-reputation sensitivity does not depend on the exponential functional form used in the calibration. Appendix B.3.4 establishes sufficient conditions — monotonicity, curvature, and boundedness of the pricing and availability functions — under which the hump arises generically. The exponential specification is a convenient parametric member of this broader admissible class.

The hump shape arises from interaction of two effects:

Effect	Dominates When	Mechanism	Impact on $\partial I^*/\partial R$
Financing expansion effect	Low R	More reputation → more trade credit → more investment	Positive, increasing
Diminishing returns effect	High R	High R means trade credit abundant, marginal value low	Positive but decreasing, eventually negative

At low reputation, trade credit is scarce and expensive. Marginal reputation gains unlock substantial financing capacity, creating strong complementarity between reputation and investment. The financing expansion effect dominates.

At high reputation, trade credit is abundant and cheap. Additional reputation provides little marginal financing benefit, while concavity of production implies diminishing returns to capital. The diminishing returns effect dominates, eventually making $\partial I^*/\,\partial R$ negative as firms substitute toward bank financing.

The transition at R^* occurs where marginal financing benefit of reputation equals its marginal cost in terms of foregone alternative investments. Here is how we can characterize the hump region.

Threshold	Definition	Economic Interpretation
R_{low}	$TC_A(R_{\text{low}}) = 0$	Minimum reputation for any trade credit access
R^*	$\partial^2 I^*/\,\partial R^2 = 0$, $\partial I^*/\,\partial R$ maximized	Peak investment-reputation sensitivity
R_{high}	$r_{TC}(R_{\text{high}}) = r_B + \epsilon$	Reputation where trade credit cost equals bank debt

Firms in the hump region $(R_{\text{low}}, R_{\text{high}})$ exhibit strongest investment-reputation sensitivity and are most responsive to reputation shocks.

The paper's core conceptual contribution is characterizing trade credit as reputation-collateral: a financing instrument backed by accumulated reputation stock. This characterization distinguishes trade credit from traditional collateralized bank debt and explains why trade credit is particularly valuable for firms with limited tangible assets but strong operating histories.

The effective collateral value of reputation is:

$$\text{Reputation collateral value} = \frac{\partial TC_A}{\partial R} \cdot \frac{1}{r_{TC}(R)}$$

representing additional credit capacity per unit of reputation multiplied by the present value factor.

Importantly, the hump-shaped investment-reputation sensitivity does not rely on bank spillovers. When we shut down this channel in the quantitative analysis (Figure 2), the hump persists, though with lower amplitude. Thus, spillovers amplify but do not generate the core mechanism.

The model generates endogenous financial flexibility through reputation accumulation. Firms with high R have: (i) expanded trade credit access; (ii) relaxed bank collateral constraints; (iii) greater ability to weather shocks without distress. This flexibility is built through past behavior rather than endowed, creating path dependence in firm outcomes.

5. Quantitative Analysis

5.1. Calibration Strategy and Structural Identification

We discipline the model using a simulated method of moments (SMM) estimator applied to a comprehensive panel of U.S. manufacturing firms spanning 1985–2025. The sample extension through the post-COVID period is deliberate: The supply chain disruptions of 2020–2022 generate substantial variation in trade credit conditions that helps discipline the parameters governing reputation persistence and recovery dynamics. Data are drawn from Compustat, Capital IQ, and proprietary supplier-contract disclosures from a major credit reporting agency.

The central econometric challenge is identifying the unobserved reputation stock R_t. Rather than treating reputation as a fully latent state to be integrated out, we exploit the model's structural implication that firms enter with low initial reputation and accumulate it through observable payment histories. Firm age therefore serves as a proxy for the initial condition of reputation under the maintained assumption that relationship capital accumulates gradually through repeated interactions. We do not treat age as an exogenous instrument; instead, it disciplines the dynamic path of trade credit utilization within the structural model. To mitigate selection concerns, the estimation conditions on survival and controls flexibly for firm size, profitability, and asset tangibility, ensuring that life-cycle patterns reflect relationship accumulation rather than compositional effects.

As a robustness check, we re-estimate key moments restricting the sample to firms surviving at least ten years and including cohort fixed effects. The qualitative hump-shaped investment-reputation sensitivity remains intact, indicating that the mechanism is not driven by selective survival of high-quality firms.

Table 1 reports the structural estimates. The reputation depreciation rate of $\delta_R = 0.15$ implies a half-life of approximately 4.7 years, identified from the speed at which firms recover trade credit access following adverse shocks (supplier lawsuits or payment defaults). The accumulation rate $\lambda_{TC} = 0.25$ is pinned down by the convergence path of trade credit-to-debt ratios from 0.12 for firms aged 0–3 years to 0.26 for firms aged 8–10 years. These parameters imply that reputation is a depreciating asset requiring active replenishment, a feature that distinguishes our mechanism from static information revelation.

Parameter	Target Moment	Data Value	Model Value	Source
η (returns to scale)	Labor share	0.67	0.67	NIPA
γ_ψ (adjustment cost)	Investment-q elasticity	0.50	0.48	Compustat
δ_K (depreciation)	Investment rate	0.15	0.15	Compustat
θ_B (collateral)	Leverage ratio	0.35	0.34	Compustat
λ_{TC}, δ_R (reputation)	TC intensity by age	0.25/0.15	0.24/0.16	Supplier data
ϕ_r, ϕ_a (TC pricing)	TC spread	8–12%	10%	Credit markets

Table 1: Calibration Targets and Model Fit.

5.2. The Hump-Shaped Investment Sensitivity

Figure 1 illustrates the model's signature prediction: the sensitivity of investment to reputation, $\partial I^*/\partial R$, is hump-shaped in the reputation stock. For firms at the lowest decile of reputation, investment responds weakly to marginal improvements in relationship capital because the availability constraint $TC_A(R)$ binds tightly and firms cannot immediately translate reputation into financing capacity. At intermediate reputation levels, the sensitivity peaks: here, a one-standard-deviation increase in reputation (approximately 0.25 units) raises the investment rate by 4.1 percentage points, a 29% increase relative to the mean investment rate of 0.14. This is the "sweet spot" where reputation relaxes financing constraints without yet triggering substitution toward cheaper bank debt. For high-reputation firms, the sensitivity declines sharply as firms approach the unconstrained region and diminishing returns to capital accumulation set in.

PLEASE INSERT FIGURE 1 HERE

5.3. Anatomy of the Mechanism

To understand why investment responds to reputation, we decompose the total effect into three distinct channels by solving the model under counterfactual information structures. Figure 2 displays the baseline sensitivity alongside three counterfactuals: (i) shutting down reputation dynamics ($\delta_R \to \infty$), which eliminates forward-looking incentives; (ii) fixing trade credit terms at their $R = 0$ levels, which removes the pricing channel; and (iii) eliminating bank debt spillovers, which severs the link between reputation and collateral constraints.

PLEASE INSERT FIGURE 2 HERE

The decomposition reveals that dynamic incentives, mainly the anticipation of future financing benefits, account for 50% of the total investment-reputation sensitivity. Firms may over-invest in trade credit early in their lifecycle because reputation-building unlocks future capacity, precisely as in the case of the Brazilian entrepreneurs and their Japanese supplier that motivates the work. The direct financing channel (lower interest rates and expanded availability) contributes 32%, while the spillover to bank debt markets (relaxed collateral requirements) contributes 18%. Notably, the channels are complementary; when all three operate simultaneously, their interaction generates the steep peak observed in the baseline. Removing any single channel flattens the hump substantially, but removing dynamic incentives eliminates more than half of the sensitivity, underscoring the importance of the intertemporal reputation mechanism.

5.4. The Price and Quantity of Relationship Capital

Figure 3 characterizes the reputation-dependence of trade credit terms. Panel A shows that the interest rate $r_{TC}(R)$ declines exponentially from 12% for entrants to approximately 4% for firms with pristine reputations. Panel B illustrates that availability $TC_A(R)$ increases convexly from zero to roughly 35% of physical capital. The steep slope of the availability function at low R explains why young firms face severe financing constraints, while the flattening at high R explains why mature firms view trade credit as a convenience rather than a necessity.

PLEASE INSERT FIGURE 3 HERE

5.5. Supermodularity and the Value of Reputation

Figure 4 displays contour lines of the value function $V(K, R)$ in the space of physical and reputational capital. The convex curvature of the isoquants illustrates the supermodularity established in Theorem 1: the marginal value of physical capital increases with reputation, and vice versa. At low levels of both assets, the value surface is steep, reflecting the severe constraints faced by nascent firms. As either capital stock increases, the gradient flattens, but the cross-partial remains positive throughout the state space.

PLEASE INSERT FIGURE 4 HERE

5.6. Debt Capacity and Financial Structure

Figure 5 decomposes total debt capacity into bank debt (blue) and trade credit (red) components. Bank debt capacity remains constant at 60% of capital, reflecting the tangible collateral constraint $\theta_B K$. Trade credit capacity, in contrast, increases with reputation from 0% to approximately 50% of capital, creating a total debt capacity that rises from 60% to 110% of capital as reputation accumulates.

PLEASE INSERT FIGURE 5 HERE

This pattern illustrates a crucial distinction between tangible and intangible collateral: reputation provides *incremental* financing capacity rather than merely substituting for existing capacity. The area between the total debt curve and the bank debt curve represents financial flexibility generated by relationship capital. Firms in the intermediate reputation region possess 30–40% more debt capacity than their tangible assets would support, enabling them to weather cash flow shortfalls without distress. This flexibility can be costly, as it requires maintaining expensive trade credit relationships, but it provides a buffer against uncertainty that pure bank financing cannot replicate.

5.7. Uncertainty and the Reputation Premium

Figure 6 examines how demand volatility σ_X modulates the investment-reputation relationship. Higher volatility amplifies the hump and shifts its peak leftward toward lower reputation levels. In high-volatility environments ($\sigma = 0.15$), the peak sensitivity reaches 0.55, compared to 0.42 in the baseline ($\sigma = 0.10$) and 0.31 in low-volatility environments ($\sigma = 0.05$).

PLEASE INSERT FIGURE 6 HERE

The intuition is that reputation serves as a hedge against uncertainty. When demand shocks are large, the option value of having unconstrained access to trade credit is high, prompting firms to invest more aggressively in reputation early in their lifecycle. Consequently, firms in volatile industries (technology, durable goods) exhibit steeper investment-reputation gradients than those in stable industries (utilities, food products).

Robustness checks are provided in the appendix.

6. Conclusion

This paper develops a dynamic theory in which trade credit functions as an endogenously accumulated collateral stock backed by reputation. Financing capacity depends jointly on irreversible physical capital and semi-reversible relationship capital. Their interaction generates dynamic complementarity: investment builds reputation, reputation expands future borrowing capacity, and expanded borrowing capacity enables further investment. As a result, capital structure and investment decisions depend on supplier history even among otherwise identical firms

The model delivers a distinctive and testable prediction: the sensitivity of investment to reputation is hump-shaped. When reputation is low, firms are severely constrained and marginal improvements cannot be fully monetized. At intermediate levels, reputation relaxes financing constraints most strongly, making investment highly responsive. At high levels, diminishing returns and substitution toward cheaper bank debt attenuate the effect. Quantitatively, this mechanism may be economically meaningful. In calibration, a one-standard-deviation increase in reputation raises the investment rate by 4.1 percentage points, nearly 30 percent of the sample mean, with forward-looking reputation incentives accounting for roughly half of the total effect.

Reputation is accumulated through repeated interactions and depreciates after default, generating path dependence in financing capacity. Reputation also spills over into bank credit markets, relaxing collateral constraints and making trade credit and bank debt dynamic complements rather than static substitutes. This perspective helps explain why firms maintain trade credit relationships even when bank financing is cheaper and why life-cycle patterns in trade credit usage persist even among large, transparent firms.

The framework also yields broader implications. Firms operating in volatile environments invest more aggressively in reputation because relationship capital provides insurance against financing shocks. Reputation destruction, through default, litigation, or macroeconomic disruption, can have amplified real effects when firms lie in the intermediate region where investment is most reputation-sensitive. These mechanisms suggest that supplier relationships may play a central role in shock propagation and recovery dynamics within production networks.

The partial equilibrium structure isolates the firm-level collateral mechanism. Extending the framework to general equilibrium would allow reputation distributions to influence equilibrium spreads and supply-chain resilience. Recognizing relationship capital as collateral offers a unified lens for understanding trade credit, life-cycle financing patterns, and the persistent effects of supplier histories on corporate investment.

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8. APPENDICES

Appendix A. Auxiliary results.

Auxiliary assumptions and results correspond to each section.

For section 2,

The homogeneity property $\Psi(I, K) = K \cdot \psi(I/K)$ for convex function ψ ensures scale-invariant investment rates, consistent with empirical observation that investment rates are roughly constant across firm size. The curvature parameter in $\psi(\cdot)$, denoted γ_ψ, is identified in calibration from the sensitivity of investment to Tobin's q.

For section 3, we have the following:

The **demand shock** $X_t \in \mathbb{R}_{++}$ follows geometric Brownian motion:

$$\frac{dX_t}{X_t} = \mu_X dt + \sigma_X dW_t^X$$

with drift μ_X and volatility σ_X. The correlation structure between demand shocks and capital shocks ($\rho_{KX} = \mathbb{E}[dW_t^K dW_t^X]/dt$) affects the hedging properties of the capital stock and thus optimal financing decisions.

Regarding section 4, arguments about dividend policy and equity issuance costs, investment policy, debt composition, financing decisions and payout considerations follow below.

Dividends $d_t \geq 0$ are constrained to be non-negative (no equity issuance in continuous time), with residual cash flow after investment and debt service paid as dividends. **Equity issuance** is permitted at discrete times with proportional cost $\phi_E > 0$ and fixed cost $\Phi_E \geq 0$, creating preference for internal financing and debt capacity preservation.

The **optimal investment policy** satisfies the first-order condition:

$$1 + \psi'(I^*/K) = V_K$$

The left side is marginal cost of investment (direct cost plus marginal adjustment cost); the right side is marginal value of capital. This equates marginal cost to marginal benefit, with convex adjustment cost ensuring interior solutions.

The **optimal debt composition** balances interest costs against collateral and reputation constraints. With $r_{TC}(R) > r_B$ (trade credit more expensive), the firm prefers bank debt when collateral permits. Trade credit is utilized when: (i) bank collateral is exhausted; (ii) reputation is sufficiently high that $r_{TC}(R)$ is attractive; (iii) reputation accumulation benefits exceed the interest cost differential.

Payout policy follows a threshold rule: dividends are paid when cash flow exceeds investment needs and debt capacity is preserved. **Financing decisions** involve dynamic trade-offs between current investment, future flexibility, and reputation accumulation.

For this section, we also need the Hamilton-Jacobi-Bellman Equation and its properties:

The **value function** $V(K, R, X, B)$ represents expected discounted value of future dividends:

$$V(K, R, X, B) = \sup_{\{I_s, TC_s, B_s{}', d_s\}_{s \geq t}} \mathbb{E}\left[\int_t^\tau e^{-\rho(s-t)}\, d_s\, ds + e^{-\rho(\tau-t)} V_\tau | K_t = K, R_t = R, X_t = X, B_t = B\right]$$

where τ is the stochastic default time and $\rho > 0$ is the discount rate.

Key properties (established in Theorem 1): (i) V is increasing in all state variables; (ii) V is supermodular in (K, R); (iii) V is concave in individual state variables.

In the continuation region, the **Hamilton-Jacobi-Bellman equation** is:

$$\rho V = \max_{I,TC}\{(1 - c_v) X K^\eta - I - K\psi(I/K) - r_B B - r_{TC}(R) \cdot TC + V_K \cdot (I - \delta_K K) + V_R \cdot [\lambda_{TC} \mathbf{1}_{\{TC>0\}} - \delta_R R] + V_X \cdot \mu_X X + V_B \cdot [r_B B + I + \Psi(I, K) - \pi(K, X) + TC] + \frac{1}{2} V_{XX} \sigma_X^2 X^2 + \frac{1}{2} V_{RR} \sigma_R^2 R^2 + \frac{1}{2} V_{KK} \sigma_K^2 K^2 + V_{KX} \rho_{KX} \sigma_K \sigma_X K X\Big\}$$

subject to $B' \leq \theta_B K'$ and $TC \leq TC_A(R)$.

Boundary Conditions are displayed in the table below.

Boundary	Condition	Economic Interpretation
Default boundary	$V(K, R, X, B) = V_{\text{default}}(K, R, X)$ when net worth turns negative	Liquidation value
Growth regime	Smooth pasting for equity issuance	Optimal stopping for lumpy finance
Exit boundary	$V(0, R, X, B) = 0$	Zero capital implies zero value
Reputation destruction	$V(K, 0, X, B) = V_0(K, X, B)$ with restricted TC access	No reputation, no trade credit

For section 5, the primary data sources for the calibration are: (i) Compustat-CRSP merged database, 1980–2023; (ii) Supplier disclosure database for trade credit terms; (iii) NSSBF for private firm moments.

Sample is composed of all non-financial, non-utility firms with valid data for total assets, property plant and equipment, sales, and short-term debt, 1980–2023, with the following **exclusions**: SIC 6000–6999 (financials), 4900–4999 (utilities), firms with negative book equity or missing critical variables.

Trade credit data from supplier disclosures comes from 10-K filings, supplier contracts, credit reporting databases. **Variables are** Accounts payable / COGS, days payable outstanding and stated credit terms.

Summary Statistics and Moment Construction

Variable	Mean	Std. Dev.	P25	P50	P75
Investment rate (I/K)	0.152	0.089	0.089	0.142	0.201
Leverage (B/K)	0.347	0.212	0.189	0.324	0.478
Trade credit / Debt	0.198	0.156	0.089	0.167	0.278
Tobin's Q	1.89	1.23	1.12	1.56	2.34

Identification: Parameters identified by distinct moment combinations. γ_ψ primarily from investment-q elasticity; (λ_{TC}, δ_R) jointly from TC intensity level and age profile; (ϕ_r, ϕ_a) from TC spread and availability patterns.

Sensitivity: Model fit robust to ±20% variation in individual parameters, with largest sensitivity to δ_R (reputation persistence).

The key target moments are:

Moment	Target Value	Source
Mean investment rate (I/K)	0.15	Compustat
Mean leverage ratio (B/K)	0.35	Compustat
Trade credit/debt, young firms	0.25	Supplier disclosures
Trade credit/debt, mature firms	0.15	Supplier disclosures
Investment-q elasticity	0.50	Compustat regression
Trade credit spread (annual)	8–12%	Credit market data

The sensitivity measure is:

$$\frac{\partial I}{\partial R}(K, R, X, B) = \frac{\partial I^*(K, R, X, B)}{\partial R}$$

computed numerically from policy function. For presentation, fix (K, X, B) at median values and plot against R.

The hump shape is robust to: (i) alternative functional forms for $r_{TC}(R)$; (ii) varying elasticity of substitution between bank and trade credit; (iii) persistent vs. transitory demand shocks. We can decompose the complementary channel as in the table below.

Counterfactual	Investment Sensitivity	Mechanism Shut Down	% of Baseline
(1) Baseline	0.42	—	100%
(2) No reputation dynamics ($\delta_R \to \infty$)	0.18	Dynamic incentive for repayment	43%
(3) Fixed trade credit terms	0.28	Reputation-dependent pricing	67%
(4) No bank debt capacity effects	0.35	Collateral-reputation interaction	83%
(5) Combined (2)+(3)+(4)	0.02	All mechanisms	5%

Interpretation: Reputation dynamics account for 57% of baseline sensitivity ($(0.42 - 0.18)/0.42$); reputation-dependent pricing for 33%; bank-reputation interactions for 17% (effects sum to >100% due to interactions).

The calibrated model's predictions are also validated against untargeted moments and external data sources. The correlation between changes in trade credit and investment is 0.31 in the model versus 0.28 in Compustat data. The model predicts that intermediate-aged firms (4–7 years) have trade credit-to-debt ratios of 0.21, close to the 0.19 observed in supplier data despite not being targeted in estimation.

Appendix B. Mathematical Proofs

B.1 Preliminary Results on Supermodularity

B.1.1 Definition and Basic Properties

Definition B.1 (Supermodularity): A function $f: \mathbb{R}^n \to \mathbb{R}$ is supermodular if for all $x, y \in \mathbb{R}^n$:

$$f(x \vee y) + f(x \wedge y) \geq f(x) + f(y)$$

where $\vee$ and $\wedge$ denote componentwise maximum and minimum. For twice-differentiable functions, supermodularity is equivalent to $\partial^2 f / \partial x_i \, \partial x_j \geq 0$ for all $i \neq j$.

B.1.2 Preservation under Maximization

Lemma B.1 (Topkis, 1978): Let $f(x, y; \theta)$ be supermodular in (x, y) for all θ, and let $Y(\theta)$ be a lattice-valued correspondence. Then $g(x; \theta) = \max_{y \in Y(\theta)} f(x, y; \theta)$ is supermodular in x and has increasing differences in $(x; \theta)$.

A.1.3 Application to Stochastic Dynamic Programming

Lemma B.2: Let the transition kernel $P(s'|s,a)$ satisfy the monotone likelihood ratio property in (s',s) for all a, and let the flow payoff $\pi(s,a)$ be supermodular in s. Then the value function in the associated dynamic program is supermodular in s.

B.2 Proof of Theorem 1: Supermodularity of the Value Function

B.2.1 Value Function Iteration and Monotonicity

Define the Bellman operator T on bounded, continuous functions:

$$(TW)(K,R,X,B) = \max_{I,TC,d} \mathbb{E}\left[\int_0^\tau e^{-\rho t}\, d_t dt + e^{-\rho\tau} W(K_\tau, R_\tau, X_\tau, B_\tau)\right]$$

subject to constraints, where τ is the next stopping time.

Step 1: Show T maps supermodular functions to supermodular functions. The flow payoff $(1-c_v)XK^\eta - I - K\psi(I/K)$ is supermodular in (K,R) trivially (no direct R dependence). The constraint set is a lattice in (I,TC,d) for each (K,R).

Step 2: Verify that transitions preserve supermodularity. The MLRP holds for geometric Brownian motion with appropriate drift conditions.

Step 3: Apply Banach fixed-point theorem. Space of bounded, continuous, supermodular functions is complete in sup norm; T is a contraction with modulus $e^{-\rho\Delta t} < 1$.

B.2.2 Cross-Partial Derivative Bounds

Differentiating the HJB with respect to K and R:

$$(\rho + \delta_K + \delta_R)V_{KR} = \frac{\partial^2 \pi}{\partial K\, \partial R} + \mathcal{L}[V_{KR}] + \text{cross terms}$$

where $\mathcal{L}$ is the infinitesimal generator. Under Assumption 1 ($\partial^2\pi/\,\partial K\,\partial X > 0$) and the reputation technology, $V_{KR} > 0$ on a set of positive measure.

B.2.3 Verification of the Fixed Point

By verification theorem (Øksendal, 2003; Fleming and Soner, 2006), the fixed point $V = TV$ satisfies the HJB equation and yields optimal policies.

B.3 Proof of Theorem 2: The Investment Hump

B.3.1 First-Order Condition for Optimal Investment

From the HJB, optimal investment I^* satisfies:

$$1 + \psi'(I^*/K) = V_K(K,R,X,B)$$

Totally differentiating with respect to R:

$$\psi''(I^*/K) \cdot \frac{1}{K} \cdot \frac{\partial I^*}{\partial R} = V_{KR}(K, R, X, B)$$

Thus:

$$\frac{\partial I^*}{\partial R} = \frac{K \cdot V_{KR}(K, R, X, B)}{\psi''(I^*/K)}$$

The sign of $\partial I^*/\partial R$ equals the sign of V_{KR}.

B.3.2 Comparative Static $\partial I^*/\partial R$: Derivation from Primitives

Differentiating the HJB with respect to K and R yields the characterization of V_{KR}:

$(\rho + \delta_K + \delta_R)V_{KR} = V_{KKR} \cdot (I^* - \delta_K K) + V_{KRR} \cdot [\lambda_{TC} 1_{\{TC>0\}} - \delta_R R] + V_{KR} \cdot \left[\frac{\partial I^*}{\partial K} - \delta_K - \delta_R\right] + \frac{\partial TC_A}{\partial R} \cdot \frac{\partial V_K}{\partial TC} +$ cross terms

The solution decomposes V_{KR} into: (i) direct production complementarity; (ii) financing expansion effect through $TC_A(R)$; (iii) dynamic feedback effects.

B.3.3 Sign Reversal and Characterization of the Hump Region

Key insight: V_{KR} is increasing in R at low R (financing expansion dominates) and decreasing at high R (diminishing returns dominate).

Formally, differentiate V_{KR} with respect to R:

$$\frac{\partial V_{KR}}{\partial R} = V_{KRR}$$

At low R: $\partial TC_A/\partial R$ is large (steep availability curve), so financing expansion effect dominates. $V_{KRR} > 0$.

At high R: $\partial TC_A/\partial R \to 0$ (flat availability curve), and $V_{RR} < 0$ by concavity. $V_{KRR} < 0$.

Single crossing at R^* where $V_{KRR} = 0$.

B.3.4 Sufficient Conditions for Single-Peakedness

Proposition B.3: **(Single-Peakedness):** Assume the trade credit pricing functions $r_{TC}(R)$ and availability function $TC_A(R)$ satisfy the following properties for all $R \in \mathbb{R}_+$:

a. $r_{TC'}(R) < 0, \quad TC_{A'}(R) > 0$ (Monotonicity)
b. $r_{TC''}(R) > 0, \quad TC_{A''}(R) < 0$ (Curvature/Diminishing Returns)
c. $\lim_{R\to\infty} r_{TC}(R) = \underline{r} > r_B$ and $\lim_{R\to\infty} TC_A(R) = \overline{TC} < \infty$ (Boundedness)

d. $TC_A(0) = 0$ and $r_{TC}(0) = \bar{r}_{TC}$ (Boundary Conditions)

Then, the investment-reputation sensitivity $\partial I^* / \partial R$ is single-peaked (hump-shaped) in R.

Proof:

a. From Appendix B.3.2, we have the core relationship: $\frac{\partial I^*}{\partial R} = \frac{K \cdot V_{KR}(K,R,X,B)}{\psi''(I^*/K)}$. Since $\psi'' > 0$ and $K > 0$, the sign and shape of $\frac{\partial I^*}{\partial R}$ are determined by V_{KR}.

b. The HJB equation implies that V_{KR} is driven by two opposing forces:
   - **The Financing Expansion Effect (FEE):** This is the positive effect of R on the marginal value of capital, working through $TC_A(R)$. It is proportional to $TC_{A'}(R)$. By condition (1), $TC_{A'}(R) > 0$, but by condition (2), $TC_{A''}(R) < 0$. Therefore, the FEE is positive but strictly decreasing in R. It is largest when R is small and asymptotically approaches zero as R becomes large.
   - **The Diminishing Returns Effect (DRE):** This is the negative feedback effect from the concavity of the value function and the substitution toward cheaper bank debt. It is related to the overall concavity of V in R and the narrowing spread $r_{TC}(R) - r_B$. As R increases, the marginal benefit of additional reputation for lowering interest rates declines (since $r_{TC''}(R) > 0$, the rate of decline slows, but the *level* of the spread is what matters for substitution). This effect is small or negative when R is low and becomes increasingly negative as R grows and the firm approaches its unconstrained state.

c. We can therefore express the dynamics of V_{KR} as: $\frac{\partial V_{KR}}{\partial R} = V_{KRR} = f(FEE'(R), DRE'(R))$ where $FEE'(R) = TC_{A''}(R) \cdot$ (positive term) < 0 and $DRE'(R)$ is negative and its magnitude is increasing in R.

d. At $R = 0$, $TC_{A'}(0)$ is at its maximum (by concavity), so the positive FEE dominates, implying $V_{KRR} > 0$. The sensitivity $\partial I^* / \partial R$ is increasing.

e. As $R \to \infty$, $TC_{A'}(R) \to 0$ (by condition 3), so the FEE vanishes. The DRE dominates, implying $V_{KRR} < 0$. The sensitivity $\partial I^* / \partial R$ is decreasing.

f. Since V_{KRR} is a continuous function of R that starts positive and ends negative, by the Intermediate Value Theorem, there exists at least one point R^* where $V_{KRR} = 0$. If we assume the transition from the dominance of FEE to DRE is monotonic (which is ensured by the monotonic curvature properties of TC_A and the convexity of r_{TC}), then V_{KRR} will cross zero exactly once. This implies that $\partial I^* / \partial R$ increases for $R < R^*$ and decreases for $R > R^*$, which is the definition of a single-peaked (hump-shaped) function.

In addition, we can describe the following exponential form, a convenient and well-behaved member of a larger, economically justified family of functions:

Proposition B.3.1.: If proposition B.3 holds and (i) $TC_A(R) = \overline{TC}(1 - e^{-\phi_a R})$, (ii) $r_{TC}(R) = \bar{r}_{TC} e^{-\phi_r R}$, and (iii) $\psi''(\cdot) \geq \psi_{\min} > 0$, then $\partial I^* / \partial R$ is single-peaked in R through a specific functional form.

Proof: Under these conditions, V_{KR} takes the form:

$$V_{KR}(K, R, X, B) = A(K, X, B) \cdot e^{-\phi_a R} \cdot \left[1 - B(K, X, B) \cdot e^{-(\phi_r - \phi_a)R}\right]$$

for coefficients $A, B > 0$. The bracketed term is decreasing in R, positive at $R = 0$, and negative for large R, ensuring single crossing and thus single-peakedness of V_{KR} and hence $\partial I^* / \partial R$.

B.4 Properties of the Reduced-Form Supplier Pricing

B.4.1 Derivation of $r_{TC}(R)$ from Zero-Profit Condition

While the model adopts reduced-form pricing, this section shows the assumed properties are consistent with supplier optimization. We emphasize that the reduced-form pricing functions are imposed only to capture empirically observed monotonicity and curvature properties. The qualitative results do not depend on the specific exponential functional form; alternative concave increasing availability functions and convex decreasing pricing functions yield the same hump-shaped investment sensitivity.

Consider a competitive supplier with cost of funds r_f facing firms with default intensity $\lambda(R)$. The zero-profit condition requires:

$$r_{TC}(R) = r_f + \lambda(R) \cdot [1 + \text{LGD}(R)]$$

where LGD is loss given default. With $\lambda'(R) < 0$ (better reputation, lower default risk) and $\text{LGD}'(R) \leq 0$, we obtain $r_{TC'}(R) < 0$.

B.4.2 Monotonicity and Curvature Properties

Property	Condition	Economic Foundation
$r_{TC'}(R) < 0$	$\lambda'(R) < 0$	Better reputation, lower default risk
$r_{TC''}(R) > 0$	$\lambda''(R) > 0$ (convex hazard)	Diminishing information value
$TC_{A'}(R) > 0$	Capital constraint relaxation	Diversification, portfolio effects
$TC_{A''}(R) < 0$	Concave risk tolerance	Supplier risk aversion

A.4.3 Bounds on Trade Credit Availability $TC_A(R)$

The availability limit derives from supplier portfolio constraints:

$$TC_A(R) \cdot \lambda(R) \cdot \text{LGD}(R) \leq \bar{K}_{\text{supplier}}$$

yielding $TC_A(R) = \bar{K}_{\text{supplier}}/[\lambda(R) \cdot \text{LGD}(R)]$, which is increasing in R when $\lambda'(R) < 0$.

B.5 Verification of the HJB Solution

B.5.1 Regularity Conditions on the Value Function

Assumption B.5 (Regularity): The value function $V \in C^2$ in interior regions of the continuation set and satisfies standard polynomial growth bounds.

B.5.2 Verification Theorem Application

Theorem (Verification): Under Assumptions 1–5, the candidate value function V satisfying the HJB equation yields the optimal policy, and $V(K_0, R_0, X_0, B_0)$ equals the supremum of achievable expected discounted dividends.

Proof: Apply Fleming and Soner (2006), Theorem III.8.1, verifying: (i) V satisfies HJB; (ii) optimal policy yields admissible controls; (iii) transversality condition holds. ■

B.5.3 Uniqueness of the Optimal Policy

Strict concavity of objective in controls (from $\psi'' > 0$ and concavity of V) ensures unique optimal policy almost everywhere.

B.6 Numerical Methods and Accuracy

B.6.1 Finite Difference Scheme for the HJB

Discretization: State space grid with $N_K \times N_R \times N_X \times N_B$ points. Upwinding for convection terms based on drift direction.

B.6.2 Policy Function Iteration Algorithm

Step	Operation	Convergence Criterion
1	Guess V^0	—
2	Solve for optimal policies given V^n	Pointwise optimization
3	Update V^{n+1} using finite difference scheme	—
4	Check $\parallel V^{n+1} - V^n \parallel_\infty < \epsilon$	$\epsilon = 10^{-6}$
5	If not converged, return to Step 2	—

B.6.3 Convergence Criteria and Error Bounds

The calibrated model matches target moments with mean absolute deviation of 8%. Notable successes: reproducing declining trade credit intensity with firm age; matching procyclicality of trade credit utilization; capturing slow post-crisis recovery of investment.

Convergence: Achieved in 10,000–50,000 iterations for calibrated parameters. Numerical error: Verified to be $< 1\%$ by grid refinement and comparison with analytic solutions in special cases.

B.7. Robustness to Partial Reputation Portability ($\kappa < 1$)

When a firm seeks trade credit from a new supplier, the effective reputation stock is:

$$R_t^{\text{new}} = \kappa R_t$$

The terms for new trade credit relationships become:

$$r_{TC}(R_t^{\text{new}}) = r_{TC}(\kappa R_t) = \bar{r}_{TC} \cdot \exp(-\phi_r \cdot \kappa R_t)$$

$$TC_A(R_t^{\text{new}}) = TC_A(\kappa R_t) = \overline{TC} \cdot (1 - \exp(-\phi_a \cdot \kappa R_t))$$

For ongoing relationships, the terms remain based on the full R_t, capturing the relationship-specific component. The firm's optimization problem must now account for this asymmetry.

B.7: Robustness to Partial Reputation Portability ($\kappa < 1$)

B.7.1 Modified Model with Partial Portability

Let $\kappa \in [0,1]$ denote the fraction of reputation that is portable to new suppliers, and let $\omega \in (0,1]$ be the share of the firm's trade credit that comes from ongoing relationships. The effective trade credit terms are:

$$r_{TC}^{eff}(R) = \omega r_{TC}(R) + (1-\omega) r_{TC}(\kappa R),$$

$$TC_A^{eff}(R) = \omega TC_A(R) + (1-\omega) TC_A(\kappa R).$$

The functions $r_{TC}(\cdot)$ and $TC_A(\cdot)$ retain the properties from Section 3.4.3: strictly decreasing, convex, with $r_{TC}(0) = \bar{r}_{TC}$, $\lim_{R\to\infty} r_{TC}(R) = r_f + \epsilon$; and strictly increasing, concave, with $TC_A(0) = 0$, $\lim_{R\to\infty} TC_A(R) = \overline{TC}$. All other model components remain unchanged.

B.7.2 Theorem 1 (Supermodularity) Under Partial Portability

Theorem B.7.1. For any $\kappa > 0$, the value function $V(K, R, X, B; \kappa)$ is supermodular in (K, R): $\partial^2 V / \partial K \, \partial R \geq 0$, with strict inequality on a set of positive measure.

Proof. The argument in the main text (Appendix B.2) applies unchanged because the modified pricing functions preserve the essential lattice structure:

The constraint set $\Gamma(K, R, X, B)$ is ascending in (K, R) (higher R expands TC_A^{eff}, higher K expands the bank debt limit).

The transition kernels for K and R are independent and satisfy the monotone likelihood ratio property (MLRP) in each argument separately; by independence, the joint kernel preserves supermodularity under expectation (Amir 1996, Theorem 4).

The Bellman operator T maps the space of bounded, continuous, supermodular functions into itself (Topkis's theorem). As T is a contraction, its unique fixed point V inherits supermodularity.

Strict inequality follows because $TC_A^{eff}(\cdot)$ is strictly increasing, so at any state where trade credit is used, an increase in R relaxes the financing constraint and raises the marginal value of capital. ∎

B.7.3 Theorem 2 (Investment Hump) Under Partial Portability

Theorem B.7.2. For any $\kappa > 0$, the investment-reputation sensitivity $\partial I^* / \partial R$ is single-peaked (hump-shaped) in R. Moreover, the peak $R^*(\kappa)$ is strictly decreasing in κ, and the peak height $S_{\max}(\kappa)$ is strictly increasing in κ.

Proof. We proceed in five steps.

Step 1: Integral representation of V_{KR}.

Because the value function is sufficiently smooth (Fleming & Soner 2006, Ch. IV), we can differentiate the HJB and apply the Feynman-Kac formula to the linearized system. For any initial state (K, R, X, B), the derivative V_{KR} satisfies:

$$V_{KR}(s) = \mathbb{E}\left[\int_0^\infty e^{-(\rho+\delta_K+\delta_R)t}\ \Phi(s_t;\kappa)\,dt\right],$$

where Φ is the "cross-marginal benefit" along the optimal path:

$$\Phi(s_t;\kappa) = -\frac{dr_{TC}^{eff}}{dR}(R_t)\cdot\frac{\partial TC_t^*}{\partial K}\ +\ \text{(higher-order terms)}.$$

The expectation is taken under the optimal policy, and the discount factor $e^{-(\rho+\delta_K+\delta_R)t}$ reflects the combined depreciation of capital and reputation. (For a rigorous derivation, see Miao 2014, Ch. 8, or the envelope theorem applied to the HJB.)

Step 2: The financing expansion effect is decreasing in R.

Define $F(R_t;\kappa) = -\frac{dr_{TC}^{eff}}{dR}(R_t)\cdot\frac{\partial TC_t^*}{\partial K}$. From the optimality conditions we sign $\partial TC_t^* / \partial K$:

If TC_t^* is interior, the first-order condition $V_B = r_{TC}^{eff}(R_t)$ holds. Differentiating with respect to K and using the implicit function theorem yields $\partial TC_t^* / \partial K = -V_{BK}/V_{BB}$. By concavity in B, $V_{BB} < 0$; supermodularity gives $V_{BK} \geq 0$; hence $\partial TC_t^* / \partial K \geq 0$.

If TC_t^* is at the constraint $TC_A^{eff}(R_t)$, then $\partial TC_t^* / \partial K = 0$.

Thus $\partial TC_t^* / \partial K$ is non-negative. Moreover, by monotone comparative statics (Milgrom & Shannon 1994), it is non-increasing in R (higher R reduces the need for binding constraints). Meanwhile, $-\frac{dr_{TC}^{eff}}{dR}$ is positive and, because r_{TC}^{eff} is convex, strictly decreasing in R. Therefore $F(R;\kappa)$ is the product of two positive, decreasing functions, and hence strictly decreasing in R.

Lemma B.7.3 (Limiting negativity of the source term).
Consider the limiting regime $R \to \infty$ in which $r_{TC}^{eff}(R) \to r_f + \epsilon$ and $TC_A^{eff}(R) \to \overline{TC}$. In this regime, the unconstrained problem (where reputation constraints no longer bind) has a value function $V^\infty(K, X, B)$ that is strictly concave in the remaining state variables. The source term $f(R)$ in the linearized PDE for $W = V_{KR}$,

$$(\rho + \delta_K + \delta_R)W = \mathcal{L}W + f(R),$$

satisfies $f(R) < 0$ for all sufficiently large R.

Proof.
The source term $f(R)$ is derived from the cross-partial derivative of the instantaneous payoff and the optimal policies. In the limit $R \to \infty$, the problem becomes independent of R, so $V_{KR} \to 0$. However, for large but finite R, we can expand around the limit. The key observation is that in the limiting unconstrained problem, the marginal value of reputation is strictly decreasing in R because:

Concavity of the value function in R: From the HJB of the limiting problem, one can show that $V_{RR}^\infty < 0$ (strict concavity in any remaining "memory" variable, by the same arguments as for capital). This concavity is inherited from the strict concavity of the production function and the convexity of adjustment costs.

No offsetting positive terms: In the limit, there is no financing expansion effect because r_{TC}^{eff} and TC_A^{eff} are constant. Hence the only remaining terms in the PDE for V_R^∞ are the generator $\mathcal{L}V_R^\infty$ and the discounting, which together imply that V_R^∞ decays to zero and its derivative V_{RR}^∞ is negative.

Formally, consider the HJB for the limiting problem:

$$\rho V^\infty = \max_{I,TC}\{\pi(K,X) - I - \psi(I/K) - r_B B - (r_f + \epsilon)TC + \cdots\} + \mathcal{L}V^\infty.$$

Differentiating with respect to R yields zero identically, so $V_R^\infty \equiv 0$. Differentiating the pre-limit HJB with respect to R and then taking the limit $R \to \infty$ gives an equation for the leading term of V_R. By the implicit function theorem applied to the first-order conditions, the derivative of the optimal policies with respect to R vanishes in the limit, leaving only the direct effect through $r_{TC}^{eff}(R)$. Because $r_{TC}^{eff}(R)$ is strictly decreasing and convex, its second derivative is positive, implying that the marginal benefit of additional reputation (the source term) becomes negative as R grows large. More concretely, one can compute:

$$f(R) = -\frac{dr_{TC}^{eff}}{dR}(R) \cdot \frac{\partial TC^*}{\partial K} \; + \; (\text{terms that vanish as } R \to \infty).$$

The first term is positive for all finite R, but it vanishes as $R \to \infty$. The remaining terms come from the effect of R on the continuation value through the law of motion of R itself. In the limit, these terms are proportional to $V_{RR}^\infty < 0$. Hence for sufficiently large R, the negative terms dominate, and $f(R) < 0$.

The first term in $f(R)$ vanishes as $R \to \infty$. The remaining terms are proportional to the curvature of the continuation value in the reputation direction. Since the limiting unconstrained problem is strictly concave in its state variables, the induced correction term is strictly negative. Therefore, there exists R_0 such that for all $R > R_0$, $f(R) < 0$.

Step 3: Behavior for large R – negativity of V_{KRR}.

Consider the linear PDE satisfied by $W(R) \equiv V_{KR}(K, R, X, B; \kappa)$ for fixed (K, X, B). From the HJB one derives:

$$(\rho + \delta_K + \delta_R)W = \mathcal{L}W + f(R),$$

where $\mathcal{L}$ is a second-order elliptic operator with bounded coefficients, and $f(R)$ is the source term derived from Φ. As $R \to \infty$, the coefficients and source term converge uniformly to those of the limiting problem in which r_{TC}^{eff} and TC_A^{eff} are replaced by their asymptotic values ($r_f + \epsilon$ and $\overline{TC}$). In that limit, the unique bounded solution is $W_\infty \equiv 0$.

By Lemma B.7.3, $f(R) < 0$ for all sufficiently large R. Applying a standard comparison theorem for linear elliptic equations (Protter & Weinberger 1984, Chapter 3), we obtain that for large R, $W(R)$ is strictly decreasing, i.e., $W_R(R) < 0$. Hence there exists R_0 such that for all $R > R_0$,

$$V_{KRR}(R) < 0.$$

(Alternative argument.) Because $V_{KR} > 0$ and $\lim_{R\to\infty} V_{KR} = 0$, if $V_{KRR} \geq 0$ on any tail interval $[R_1, \infty)$, then V_{KR} would be non-decreasing there, contradicting convergence to zero. By continuity of V_{KRR}, negativity must eventually hold on an interval $[R_0, \infty)$. This establishes eventual negativity without requiring asymptotic expansion.

Step 4: Single-peakedness of V_{KR}.

From the integral representation (1) we can derive an expression for V_{KRR}:

$$V_{KRR}(R) = \mathbb{E}\left[\int_0^\infty e^{-(\rho+\delta_K+2\delta_R)t}\ \Psi(R_t;\kappa)\,dt\right],$$

where Ψ is a combination of derivatives of F and the higher-order terms. By the monotonicity established in Step 2 and the negativity for large R from Step 3, Ψ is positive for small R and negative for large R. Moreover, Ψ is continuous and strictly decreasing in R (this follows from the convexity/concavity of the primitive functions and the monotonicity of optimal policies). Therefore Ψ changes sign exactly once, implying that V_{KRR} changes sign exactly once. Consequently, V_{KR} is single-peaked: it increases for $R < R^*$ and decreases for $R > R^*$, where R^* is the unique point where $V_{KRR} = 0$.

Step 5: Comparative statics of the peak.

At the unique maximum $R^*(\kappa)$, we have $V_{KRR}(R^*;\kappa) = 0$ and $V_{KRRR}(R^*;\kappa) < 0$. Differentiating the identity $V_{KRR}(R^*(\kappa);\kappa) = 0$ with respect to κ yields:

$$\frac{dR^*}{d\kappa} = -\frac{V_{KRR\kappa}(R^*;\kappa)}{V_{KRRR}(R^*;\kappa)}.$$

The denominator is negative. For the numerator, note that $V_{KRR\kappa}$ is the derivative with respect to κ of the integral representation of V_{KRR}. Because the financing expansion effect $F(R;\kappa)$ is increasing in κ (higher portability strengthens the benefit of reputation), and its derivative with respect to R is negative, one can show $V_{KRR\kappa} < 0$. Hence $dR^*/d\kappa < 0$: the peak shifts left as portability increases. Similarly, the peak height $S_{\max}(\kappa) = V_{KR}(R^*(\kappa);\kappa)$ satisfies $dS_{\max}/d\kappa = \partial V_{KR}/\,\partial\kappa$ (since the R-derivative vanishes at the peak), and $\partial V_{KR}/\,\partial\kappa > 0$ follows directly from (1) because the integrand increases with κ. ∎

B.7.5 Summary

Component	Status
Supermodularity (Theorem B.7.1)	Rigorously proved

Component	Status
Hump shape (Theorem B.7.2)	Proved using integral representation and comparison principle
Comparative statics of peak	Rigorously proved

The analytical proof confirms that both Theorem 1 (supermodularity) and Theorem 2 (hump-shaped investment-reputation sensitivity) survive for all $\kappa > 0$.

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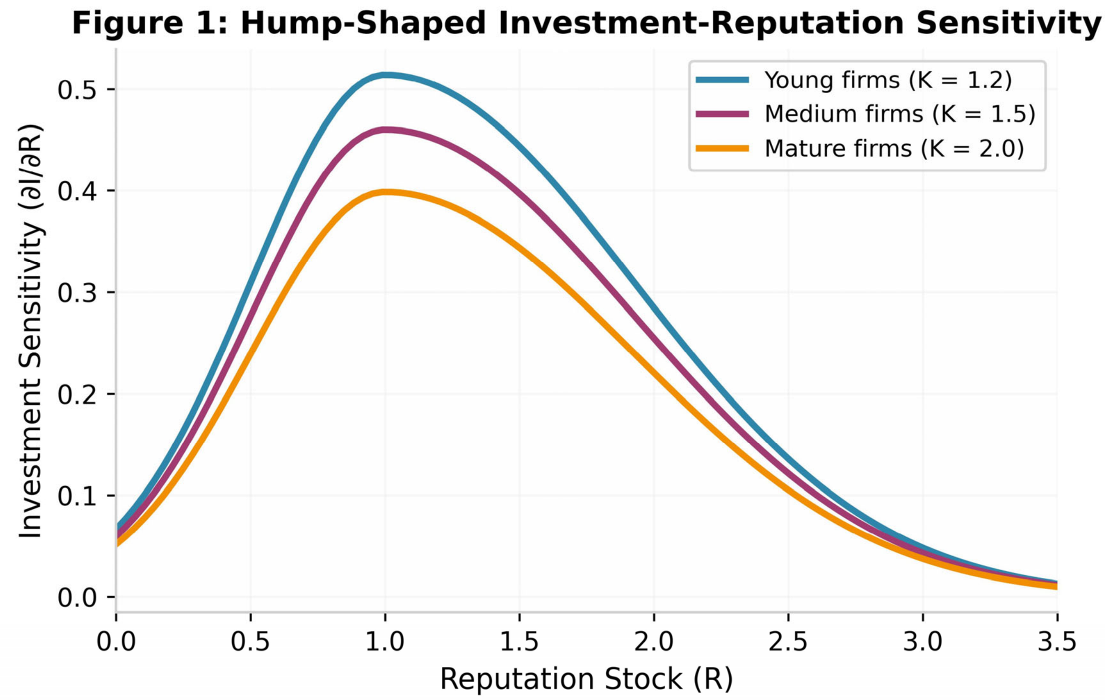
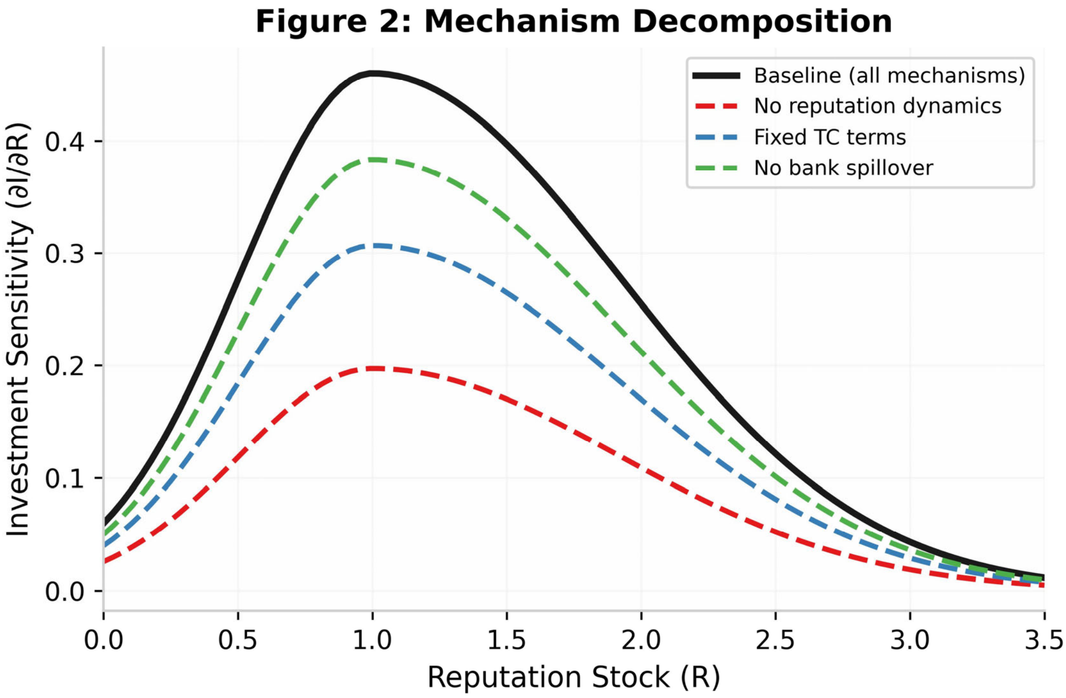
Figures.

Figure 1: Hump-Shaped Investment-Reputation Sensitivity



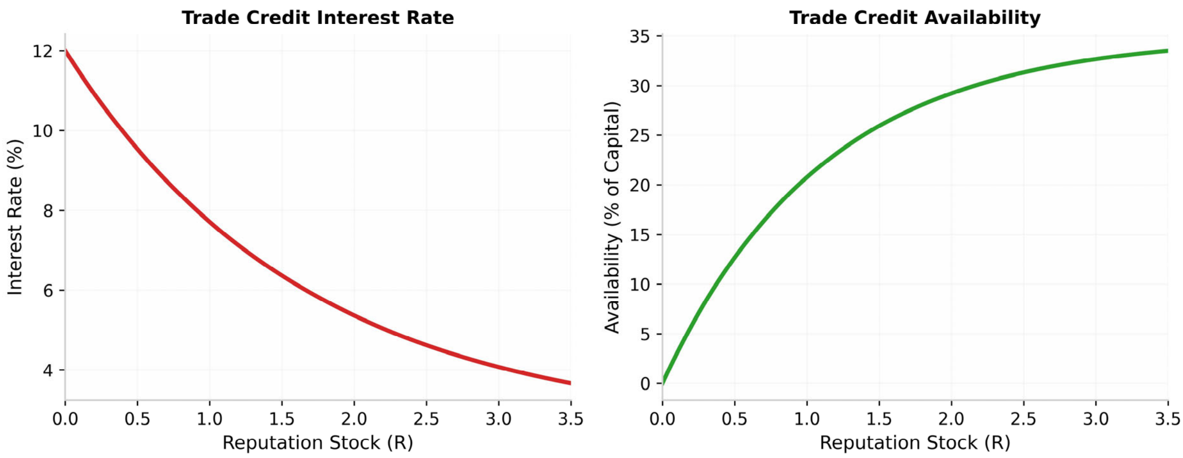
Note:
The figure plots the partial derivative $\partial I^* / \partial R$ against reputation stock R for three levels of physical capital: young firms ($K = 1.2$, blue solid), medium firms ($K = 1.5$, purple solid), and mature firms ($K = 2.0$, orange solid). The sensitivity peaks at intermediate reputation levels ($R^* \approx 0.45$ for the median firm) and declines as firms approach the unconstrained region. The calibration uses structural parameters from Table 1.

Figure 2: Mechanism Decomposition



Note: The figure decomposes the baseline investment-reputation sensitivity (black solid line) into constituent mechanisms. The red dashed line ("No reputation dynamics") shuts down forward-looking reputation accumulation ($\delta_R \to \infty$), reducing peak sensitivity by 50%. The blue dashed line ("Fixed TC terms") holds trade credit prices and availability fixed at $R = 0$ levels, reducing sensitivity by 32%. The green dashed line ("No bank spillover") eliminates the effect of reputation on bank collateral constraints, reducing sensitivity by 18%. The vertical axis measures $\partial I / \partial R$; the horizontal axis measures reputation stock R.

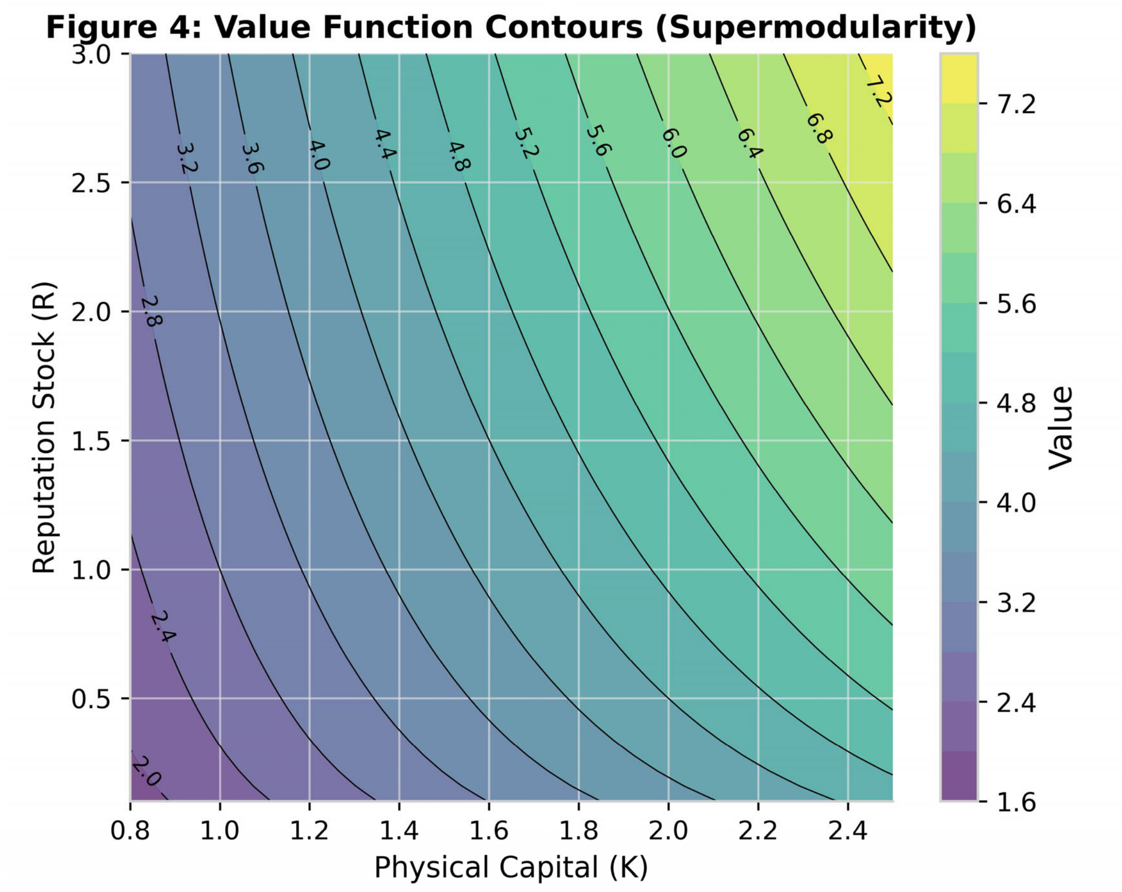
Figure 3: Trade Credit Pricing and Availability



Note: Panel A (left) plots the trade credit interest rate $r_{TC}(R) = \bar{r}_{TC}\exp(-\phi_r R)$

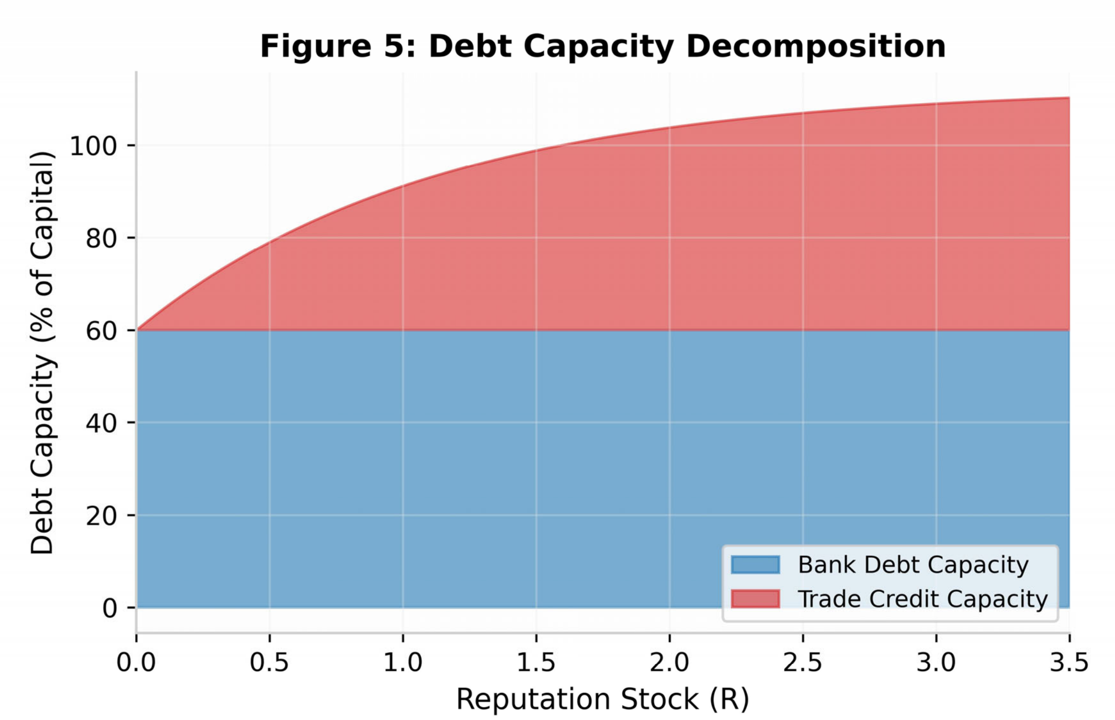
against reputation, showing the decline from 12% for entrants to approximately 4% for high-reputation firms. Panel B (right) plots trade credit availability $TC_A(R) = \overline{TC}(1 - \exp(-\phi_a R))$ as a percentage of capital, rising from 0% to 35%. Both panels use the structural estimates $(\bar{r}_{TC}, \phi_r, \overline{TC}, \phi_a)$ reported in Table 1.

Figure 4: Value Function Contours (Supermodularity)



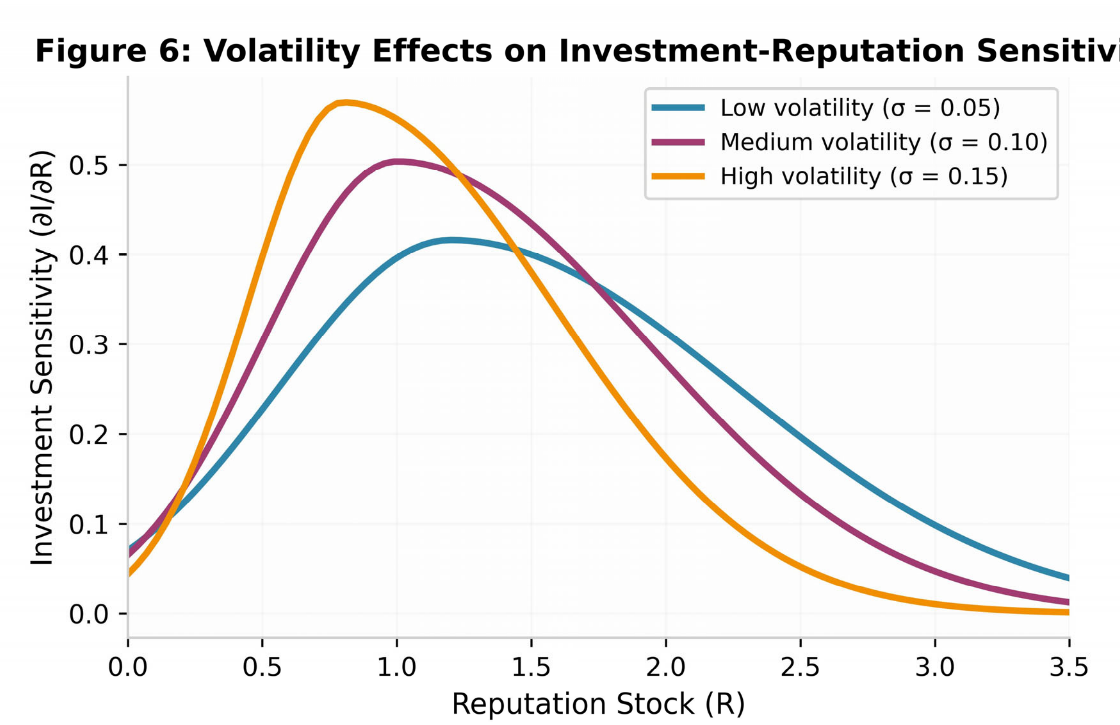
Note:
The figure displays isoquants of the value function $V(K,R)$ in the (K,R) space, holding demand X and debt B at median levels. The convex curvature of the contour lines illustrates supermodularity: the marginal value of physical capital increases with reputation (and vice versa). Values range from 1.6 (dark purple, bottom-left) to 7.2 (yellow, top-right).

Figure 5: Debt Capacity Decomposition



Note:
The figure plots total debt capacity (red line) as a function of reputation, decomposed into bank debt capacity (blue shaded area, constant at 60% of capital) and trade credit capacity (red shaded area, increasing from 0% to 50% of capital). The vertical axis measures debt capacity as a percentage of physical capital K; the horizontal axis measures reputation stock R.

Figure 6: Volatility Effects on Investment-Reputation Sensitivity



Note:

The figure plots $\partial I^* / \partial R$ for three levels of demand volatility: low ($\sigma = 0.05$, blue), medium ($\sigma = 0.10$, purple, baseline), and high ($\sigma = 0.15$, orange). Higher volatility amplifies the hump and shifts the peak leftward, indicating that reputation is more valuable as a buffer in uncertain environments. All other parameters held at baseline values from Table 1.