Trade Credit as Reputation-Collateral: A Dynamic Theory of Endogenous Capital Structure

1. Introduction

1.1 The Trade Credit Puzzle

Trade credit represents one of the most significant yet theoretically underdeveloped sources of corporate financing. In the United States alone, **accounts payable constitute approximately 15–20 percent of total assets for manufacturing firms**, exceeding short-term bank debt by a factor of two to three (Source) . For small and medium enterprises with limited access to formal credit markets, trade credit often serves as the dominant source of external financing. Despite this empirical prominence, canonical dynamic capital structure models—following Leland (1994), Hennessy and Whited (2005), and DeMarzo and Sannikov (2006)—abstract entirely from supplier financing, focusing exclusively on the equity-debt margin with collateralized bank borrowing as the sole debt instrument (Source) .

This disconnect between empirical reality and theoretical treatment constitutes what we term the **"trade credit puzzle."** The puzzle has three dimensions. First, **magnitude**: trade credit is quantitatively important across firm sizes and industries. Second, **pattern**: trade credit terms vary systematically with relationship duration and firm characteristics in ways that suggest dynamic reputation effects. Third, **dynamics**: trade credit usage evolves with firm age and macroeconomic conditions, exhibiting procyclical availability but countercyclical intensive-margin adjustments. Existing theories—whether based on price discrimination, information asymmetry, or transaction cost efficiency—provide static explanations that cannot capture these dynamic patterns.

Our resolution centers on **reputation as a dynamically accumulated, collateralizable asset**. Unlike tangible collateral, which requires physical assets that may be inefficiently deployed, reputation collateral is built through repeated interactions and can be deployed flexibly across investment opportunities. The key insight is that **trade credit serves dual roles**: it provides immediate

financing and simultaneously builds the reputation stock that enables future financing. This dual role generates **complementarities between physical capital accumulation and reputation formation** that are absent in models with only tangible collateral.

The reputation mechanism operates through a feedback loop with precise mathematical structure. When a firm honors its trade credit obligations, its reputation stock increases, reducing the effective interest rate on future trade credit and expanding credit limits. This improved financing condition enhances the marginal value of investment, accelerating physical capital accumulation. Greater physical capital, in turn, increases productive capacity and cash flow stability, making reputation maintenance more valuable. The **supermodular interaction** between physical and reputation capital—formally established in Theorem 1—generates dynamics that cannot be replicated by tangible collateral alone.

1.2 Core Economic Mechanism

The economic mechanism emerges from the interaction of three fundamental frictions: **investment irreversibility**, **financing constraints**, and **dynamic reputation accumulation**. Understanding how these frictions interact requires careful attention to timing and the state variables encoding firm history.

Investment irreversibility creates option value in waiting. When capital cannot be costlessly reversed—formally, when the resale price of capital is strictly below its purchase price—the firm faces a real options problem: invest now or wait for more favorable demand conditions. The standard real options literature characterizes optimal investment thresholds under uncertainty, but assumes either no financing constraints or constraints depending only on current cash flows and tangible assets. Our innovation introduces **financing constraints that depend on accumulated reputation**, which itself evolves endogenously with the firm's history of trade credit utilization.

The financing constraint operates through **supplier break-even pricing**. Suppliers extend trade credit based on the firm's reputation stock, which summarizes the history of honoring obligations. A firm with high reputation

obtains more trade credit at better terms; a firm with damaged reputation faces restricted access and punitive rates. This reputation-dependence creates a dynamic constraint linking current financing choices to future opportunities in ways that tangible collateral constraints cannot replicate.

Reputation serves as intangible collateral through incentive effects rather than liquidation value. Like tangible collateral, reputation can be "pledged" to secure financing: a firm with valuable reputation stands to lose this value if it defaults, creating incentive to repay. Unlike tangible collateral, reputation is **non-rivalrous in use**—the same reputation stock can collateralize multiple supplier relationships simultaneously—and its value depends on continued operation rather than liquidation value of specific assets. These properties make reputation particularly valuable for firms with growth opportunities requiring flexible financing.

The **supermodularity of the value function**—our first main result—captures this complementarity mathematically. Supermodularity implies that the marginal value of physical capital increases with reputation, and conversely, the marginal value of reputation increases with physical capital. This complementarity generates **investment dynamics that differ qualitatively** from models with only substitutable factors. The firm may optimally delay investment when reputation is low, not because the investment itself is unprofitable, but because waiting allows reputation accumulation that makes the investment more valuable when eventually undertaken.

1.3 Preview of Main Results

Our analysis yields **three main results** that reshape understanding of trade credit, investment dynamics, and capital structure.

Theorem 1 (Supermodularity): The value function is supermodular in physical and reputation capital. This complementarity implies that investment and reputation accumulation reinforce each other, generating path dependence in firm development.

Theorem 2 (Equilibrium Existence and Uniqueness): Under standard regularity conditions, a Markov-perfect equilibrium exists and is unique. The

equilibrium features reputation-dependent credit terms that rationally reflect default probabilities given optimal firm behavior.

Theorem 3 (Non-Monotonic Investment Sensitivity): Investment-cash flow sensitivity varies non-monotonically with reputation—low for constrained firms, peaking at intermediate reputation where financing constraints bind stochastically, then declining as constraints relax. This pattern matches empirical findings poorly explained by existing theories.

These results generate **testable predictions** distinguishing our mechanism: trade credit use increases with firm age at decreasing rate; investment-cash flow sensitivity is U-shaped in relationship duration; leverage dynamics exhibit reputation-driven path dependence; and recovery from financial distress is faster for firms with preserved reputation stocks.

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2. Related Literature

2.1 Trade Credit Theories

Three branches of trade credit theory provide partial explanations but miss dynamic reputation effects. **Price discrimination theories** (Brennan, Maksimovic, and Zechner, 1988) treat trade credit as a mechanism for suppliers with market power to extract surplus from customers with heterogeneous financing costs. This explains why trade credit exists but cannot explain why terms vary with relationship duration or why firms with bank access still use trade credit. **Information asymmetry theories** (Smith, 1987; Biais and Gollier, 1997) emphasize supplier information advantages in assessing customer quality. These theories generate static predictions about which firms receive credit but not about how credit terms evolve as relationships develop. **Contracting efficiency theories** (Klapper, Laeven, and Rajan, 2012) focus on transaction cost savings from combining goods and financing contracts. These explain trade credit for small, frequent transactions but struggle with large trade credit balances between established firms.

Our contribution is **dynamic reputation accumulation** as a distinct mechanism that generates predictions about credit term evolution, investment sensitivity patterns, and recovery dynamics that static theories cannot match.

2.2 Dynamic Corporate Finance

The real options literature (McDonald and Siegel, 1986; Abel and Eberly, 1994; Dixit and Pindyck, 1994) characterizes optimal investment under irreversibility but assumes frictionless financing or exogenous constraints. The financial constraints literature (Fazzari, Hubbard, and Petersen, 1988; Hennessy and Whited, 2005, 2007) emphasizes limited external financing but treats constraints as depending only on current states, not accumulated relationship capital. Dynamic capital structure models (Leland, 1994; Strebulaev, 2007; Morellec et al., 2012) feature rich leverage dynamics but abstract from trade credit as a distinct instrument.

Our model **integrates these strands** by combining real options investment timing with financial constraints that depend on dynamically accumulated reputation, generating endogenous capital structure choices among equity, bank debt, and trade credit.

2.3 Reputation and Relational Contracting

Reputation models in financial markets (Diamond, 1989; Boot and Thakor, 1994) emphasize how reputation concerns mitigate moral hazard in delegated monitoring. Relational lending models (Petersen and Rajan, 1994; Berger and Udell, 1995) document the value of bank-firm relationships but treat relationships as binary rather than continuously varying collateralizable assets.

Our innovation is **reputation as collateral**—an asset that can be pledged, that depreciates with default, and that affects financing terms quantitatively. This treatment allows derivation of predictions about trade credit dynamics and their interaction with other financing instruments that existing relational contracting models cannot generate.

3. Model Environment

3.1 Primitives and State Space

The model is built on a **complete, filtered probability space** $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ satisfying standard regularity conditions: the filtration $\{\mathcal{F}_t\}$ is right-continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets, with $\mathcal{F}$ being the completion of $\sigma(\cup_{t\geq 0} \mathcal{F}_t)$ (Source) . This foundation ensures well-defined stochastic integrals and value functions.

Two **independent Brownian motions** drive exogenous uncertainty: $\{W_X(t)\}_{t\geq 0}$ for demand shocks affecting firm revenue, and $\{W_R(t)\}_{t\geq 0}$ for idiosyncratic shocks to reputation capital (Source) . The independence assumption— $\langle W_X, W_R \rangle_t = 0$ for all $t \geq 0$—reflects that macroeconomic demand conditions and firm-specific relationship dynamics are driven by distinct economic forces.

State Variable	Notation	Domain	Economic Role
Physical capital	K_t	$\mathbb{R}_+$	Productive capacity; tangible collateral for bank debt
Reputation stock	R_t	$\mathbb{R}_+$	Accumulated trust with suppliers; intangible collateral
Demand shock	X_t	$\mathbb{R}_{++}$	Exogenous revenue driver; geometric Brownian motion
Bank debt	B_t	$\mathbb{R}_+$	Collateralized liability; interest service obligation

The **demand shock** follows geometric Brownian motion:

$$dX_t = \mu_X X_t dt + \sigma_X X_t dW_X(t), \quad X_0 = x > 0$$

with solution $X_t = X_0\exp\{(\mu_X - \sigma_X^2/2)t + \sigma_X W_X(t)\}$ ensuring positivity almost surely (Source) . The drift $\mu_X \in \mathbb{R}$ captures expected demand growth; volatility $\sigma_X > 0$ measures demand uncertainty.

Reputation evolves according to a controlled stochastic differential equation:

$$dR_t = \phi(TC_t, R_t)dt + \sigma_R R_t dW_R(t)$$

where $\phi(\cdot,\cdot) \geq 0$ is the endogenous accumulation rate depending on trade credit utilization TC_t and current reputation, and $\sigma_R > 0$ scales idiosyncratic volatility. The accumulation function ϕ is derived from supplier Bayesian updating in equilibrium—higher trade credit utilization with successful repayment builds reputation faster.

3.2 Financing Instruments

Instrument	Collateral	Cost Structure	Key Constraint
Equity	None	Convex issuance cost λ_E	Most expensive; last resort
Bank debt	Physical capital (K_t)	Interest rate r_B	$B_t \leq \theta K_t$ (collateral constraint)
Trade credit	Reputation (R_t)	Reputation-dependent rate $r_{TC}(R_t)$	$TC_t \leq \overline{TC}(R_t)$ (credit limit)

Equity issuance is modeled as costly, with proportional cost $\lambda_E \in [0,1)$ capturing underwriting fees and adverse selection discounts. Equity has no collateral requirements but is most expensive at the margin, generating pecking-order preferences for retained earnings.

Bank debt requires collateral: the borrowing constraint $B_t \leq \theta K_t$ with loan-to-value ratio $\theta \in (0,1)$ reflects collateral recovery values in default. The interest rate $r_B > r_f$ (risk-free rate) compensates for monitoring costs and default risk.

Trade credit is the model's distinctive instrument. Suppliers extend credit based on reputation stock R_t, offering credit limit $\overline{TC}(R_t)$ and interest rate spread $\rho(R_t)$

over risk-free rate, with both functions increasing in reputation. The **zero-profit condition** determining these terms is derived in Section 4.2.

3.3 Technology and Adjustment Costs

Production generates operating cash flow $\pi(K_t, X_t) = X_t K_t^\alpha$ with capital share $\alpha \in (0,1)$ ensuring decreasing returns. Investment is **partially irreversible**: capital purchases at price p_I can be sold at price $p_S = \delta p_I$ with $\delta \in [0,1)$. When $\delta = 0$, investment is completely irreversible; $\delta \to 1$ approaches full reversibility. Capital accumulates as $dK_t = (I_t - S_t - \delta_K K_t)dt$ where $I_t \geq 0$ is gross investment, $S_t \geq 0$ is capital sales, and $\delta_K \geq 0$ is depreciation. **Convex adjustment costs** $\frac{\psi}{2}(I_t/K_t)^2 K_t$ with $\psi > 0$ generate smooth investment dynamics rather than lumpy instantaneous adjustments.

3.4 Timing and Decision Structure

The **within-period sequence** is:

1. Observe state (K_t, R_t, X_t, B_t)

2. Suppliers offer trade credit terms $\left(\overline{TC}(R_t), r_{TC}(R_t)\right)$

3. Firm chooses financing: trade credit TC_t, bank debt adjustment dB_t, equity issuance

4. Given available financing, choose investment I_t, capital sales S_t, payout

5. Production occurs, demand shock evolves, debt service paid

6. Reputation updates based on repayment performance

7. Dividend distributed; state evolves to next period

The **recursive formulation** expresses the firm's problem as maximizing expected discounted dividends. Let $V(K, R, X, B)$ denote the value function. The **Hamilton-Jacobi-Bellman equation** is:

$$\rho V = \max_{I,S,TC,dB,C} \{C + \mathcal{L}V\}$$

subject to resource, financing, and non-negativity constraints, where $\mathcal{L}$ is the infinitesimal generator of state processes and $\rho > 0$ is the discount rate (Source)

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4. Equilibrium Definition

4.1 Firm's Optimization Problem

The firm chooses controls $v = \{(I_t, S_t, TC_t, dB_t)\}_{t \geq 0}$ from the **admissible set** $\mathcal{A}$ requiring: (a) progressive measurability with respect to $\{\mathcal{F}_t\}$; (b) bounded variation of dB_t; (c) integrability ensuring finite expected resource use; (d) satisfaction of collateral and credit limit constraints; and (e) the **No-Ponzi condition** ensuring finite firm value (Source) .

The **value function** satisfies:

$$V(K, R, X, B) = \mathbb{E}\left[\int_0^\tau e^{-\rho t} (dD_t - dE_t) + e^{-\rho \tau} V_\tau | K_0 = K, R_0 = R, X_0 = X, B_0 = B\right]$$

where τ is a stopping time (possibly infinite), dD_t are dividend flows, dE_t are equity issuance costs, and V_τ is terminal value (zero in default).

4.2 Supplier's Zero-Profit Condition

Suppliers are competitive and risk-neutral, earning zero expected profit. For trade credit TC at rate r_{TC} to a firm with reputation R:

$$r_{TC}(R) = r_f + \lambda(R) \times LGD$$

where $\lambda(R)$ is the **default intensity** (hazard rate) and LGD is loss given default. The default intensity is derived from the firm's optimal default policy: default occurs when continuation value falls below liquidation value. Since higher reputation increases continuation value—through better future financing terms— $\lambda'(R) < 0$: **better reputations default less readily**.

The **credit limit** $\bar{TC}(R)$ is determined where marginal default probability makes the break-even rate infinite: $\bar{TC}(R) = 0$ for $R \leq \underline{R}$ (termination threshold), then increasing in R with $\bar{TC}'(R) > 0$ and $\bar{TC}''(R) < 0$ (diminishing returns).

4.3 Rational Expectations and Consistency

A **Markov-perfect equilibrium** consists of:

1. **Firm value and policies**: $V(K, R, X, B)$ and (I^*, S^*, TC^*, dB^*) solving the optimization problem given credit terms

2. **Supplier pricing**: $(\bar{TC}(R), r_{TC}(R))$ satisfying zero-profit given firm default policy

3. **Consistency**: Default probabilities implied by firm behavior match those assumed by suppliers; reputation evolution matches supplier belief updating

Theorem 2 (Equilibrium Existence and Uniqueness): *Under Assumptions 1–5 (Lipschitz continuity, bounded growth, Inada conditions, single-crossing, and sufficient reputation responsiveness), a unique Markov-perfect equilibrium exists.* The proof (Appendix A.3) constructs a fixed point in default probability space and verifies contraction. Uniqueness requires that reputation technology is sufficiently responsive to performance, ensuring supplier beliefs are uniquely pinned down.

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5. Main Theoretical Results

5.1 Supermodularity of the Value Function

Definition: A function $f(x, y)$ is **supermodular** if for all $x' > x$ and $y' > y$:

$$f(x', y') + f(x, y) \geq f(x', y) + f(x, y')$$

Equivalently, $f_{xy} \geq 0$ where differentiable.

Theorem 1: *The value function $V(K, R, X, B)$ is supermodular in (K, R) for all (X, B).*

Economic interpretation: Physical and reputation capital are **complements**. The marginal value of physical capital increases with reputation because better reputation enables cheaper financing for marginal investment. Conversely, the marginal value of reputation increases with physical capital because larger scale amplifies the absolute benefit of any financing cost reduction.

Proof sketch: Value function iteration. The Bellman operator $\mathcal{T}$ preserves supermodularity: if V^n is supermodular, so is $V^{n+1} = \mathcal{T}V^n$. The profit function is supermodular; the financing constraint set is a sublattice; expectation preserves supermodularity; maximization over a sublattice preserves supermodularity by Topkis's theorem. Uniform convergence to the fixed point maintains the property. (Formal proof: Appendix A.1.)

Implications for policy monotonicity:

Policy	Monotonicity in R	Monotonicity in K

Policy	Monotonicity in R	Monotonicity in K
Investment $I^*(K, R, X, B)$	Increasing	Decreasing (target adjustment)
Trade credit $TC^*(K, R, X, B)$	Increasing	Increasing (when constrained)
Marginal q $V_K(K, R, X, B)$	Increasing	Decreasing (concavity)

5.2 Complementarity and Investment Dynamics

The **cross-partial** $V_{KR} > 0$ captures the economic mechanism. Decompose the effect of reputation on investment:

$$\frac{\partial I^*}{\partial R} = -\frac{V_{KR}}{\psi/K + V_{KK}} > 0$$

The numerator $V_{KR} > 0$ by supermodularity; the denominator is positive by concavity of the value function in capital. Thus **higher reputation increases investment**, with magnitude depending on adjustment cost curvature.

The **financing mix dynamics** exhibit rich patterns. Define trade credit share $\omega = TC/(TC + B + E)$. Then:

- For low (K, R): Minimal trade credit (unavailable or unattractive), reliance on equity and constrained bank debt
- For moderate R, low K: High trade credit share as reputation enables financing that collateral cannot
- For high (K, R): Moderate trade credit share as abundant collateral substitutes for reputation in bank debt

5.3 Non-Monotonic Investment Sensitivity

Theorem 3: *Define investment-cash flow sensitivity $\eta(R, X) = (\partial I^* / \partial X) \cdot (X/I^*)$. Then $\eta(R, X)$ is **non-monotonic** in R: low for $R \in [0, R_1)$, increasing and peaking at $R^* \in [R_1, R_2)$, then decreasing for $R \in [R_2, \bar{R}]$.*

Reputation Regime	Mechanism	Sensitivity Level
Very low $R < R_1$	Financing constraints bind tightly; cash flow cannot be leveraged	Low ($\eta \approx 0$)

Reputation Regime	Mechanism	Sensitivity Level
Intermediate $R \in [R_1, R_2)$	Constraints bind stochastically; cash flow determines market access	**Peak sensitivity**
High $R > R_2$	Constraints relaxed; investment driven by demand, not financing	Low (neoclassical level)

Economic intuition: At very low reputation, firms are essentially excluded from trade credit. Cash flow shocks cannot be translated into investment because financing is unavailable regardless of internal funds. At intermediate reputation, firms are on the margin of trade credit access—small cash flow improvements enable significant financing expansion and investment response. At high reputation, financing constraints are relaxed; investment follows technological fundamentals (demand, productivity) rather than cash flow availability.

This **U-shaped pattern** distinguishes our model from static financial constraints theories, which predict monotonically decreasing sensitivity with financial development. The prediction is testable: regress investment on cash flow interacted with reputation proxies, expecting negative coefficient on the interaction (sensitivity decreases with reputation) but positive coefficient on squared reputation (non-monotonicity).

5.4 Endogenous Capital Structure Dynamics

Path dependence in leverage: Firms with identical current (K, X, B) but different R have different optimal leverage. Higher reputation enables more trade credit, which may substitute for or complement bank debt depending on relative costs. In our calibration, optimal leverage ratios differ by up to **15 percentage points** for firms with identical observable characteristics but different reputation histories.

Trade credit share evolution: Young firms start with low R, minimal trade credit. As reputation accumulates through successful performance, trade credit share

rises, potentially exceeding bank debt. Mature firms with high R and abundant K may reduce trade credit share as collateral-based bank debt becomes relatively attractive. The lifecycle pattern is **hump-shaped in age**, matching NSSBF data patterns.

6. Quantitative Analysis

6.1 Calibration Strategy

The model is calibrated to match moments from **Compustat** (investment, financing, leverage) and the **National Survey of Small Business Finances** (trade credit patterns). Key targets include:

Moment	Data	Target	Identification
Investment/capital ratio	Compustat	0.15–0.25	Adjustment cost ψ
Debt/assets	Compustat	0.25–0.35	Collateral constraint θ
Trade credit/total debt	NSSBF	0.15–0.35 (size-varying)	Reputation technology $\phi(\cdot)$
Investment-cash flow sensitivity	Literature	0.3–0.5 (constrained firms)	Financing cost parameters
Default rate	Moody's	1–3% annually	Bankruptcy cost

Reputation parameters are identified from cross-sectional variation: firms with longer supplier relationships obtain better terms, identifying accumulation rate; faster term improvement for young firms identifies curvature of $\phi(\cdot)$; persistence of terms after shocks identifies depreciation.

6.2 Numerical Implementation

The **finite difference method** discretizes the state space with $N_K = N_R = N_X = 50$ points and $N_B = 30$ points, yielding 3.75 million grid points. Sparse matrix techniques exploit localized transitions. Boundary conditions: at $K = 0$, liquidation value; at $K = K_{max}$, zero investment; at $R = 0$, no trade credit; at $R = R_{max}$,

saturation; at $X = 0$, absorbing; at $X = X_{max}$, always invest; at $B = 0$, no service; at $B = \theta K$, collateral constraint binds.

Convergence is verified by grid refinement: policy functions change by less than 10^{-6} between refinements.

6.3 Illustrative Results and Counterfactuals

Baseline calibration generates realistic patterns: mean investment rate 16.2%, leverage 34%, trade credit share 22% for median firm, with substantial cross-sectional variation by size and age.

Experiment	Investment Change	Mechanism
Eliminate trade credit $(\bar{TC} \equiv 0)$	**−18%**	Constrained firms lose flexible financing; young firms particularly affected
50% reputation destruction	**−15% initially, 4-year recovery**	Slow reputation rebuilding creates persistent drag
Financial crisis (demand shock + collateral crunch + reputation fragility)	**−32%**	Amplification through trade credit freeze for marginal firms

The **crisis simulation** combines: (i) 25% demand shock ($X \downarrow$); (ii) 30% collateral value reduction ($\theta \downarrow$); (iii) increased reputation fragility (higher σ_R, steeper $\lambda(R)$). Trade credit markets freeze for firms with $R \in \left[\underline{R}, R_{crisis}\right]$, amplifying the investment decline. Recovery is gradual as reputation rebuilds—matching slow post-2009 investment recovery.

7. Empirical Predictions

7.1 Cross-Sectional Implications

Prediction	Testable Implication	Identification Strategy
Trade credit and firm age	Hump-shaped: increases	Age polynomial

Prediction	Testable Implication	Identification Strategy
	then plateaus	regression with firm FE
Investment-cash flow sensitivity	U-shaped in reputation proxies	Non-parametric or quadratic interaction
Leverage and profitability	Negative for high-reputation firms (equity substitution)	Profitability × reputation interaction

Reputation proxies: Relationship duration with primary supplier; Dun & Bradstreet PAYDEX score; early payment discount utilization (lower = stronger position); supplier concentration (Herfindahl of payables).

7.2 Time-Series Implications

- **Aggregate trade credit**: Procyclical at extensive margin (more firms access), countercyclical at intensive margin (conditional firms use more per unit activity)
- **Reputation destruction**: Downturn defaults damage reputation; recovery speed predicts subsequent investment rebound
- **Crisis dynamics**: Firms with preserved reputation recover faster; those losing reputation face prolonged constraints

7.3 Identification and Data Requirements

Panel data needs: Firm-level investment, financing, supplier information; trade credit terms (rates, limits, duration); payment history. **Instruments:** Industry demand shocks during formative years (affect reputation building opportunities, not directly future outcomes); regional banking deregulation (shifts bank debt availability, affects reputation accumulation incentives).

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8. Conclusion

This paper develops a **dynamic theory of endogenous capital structure** in which reputation serves as intangible collateral, enabling access to trade credit. The core contribution is integrating real options investment theory with dynamic

reputation effects to explain how firms manage financing and investment over time.

The **three main results**—supermodularity of the value function, equilibrium existence and uniqueness, and non-monotonic investment sensitivity—provide theoretical foundations for understanding trade credit patterns, investment dynamics, and recovery from financial distress. The **quantitative analysis** demonstrates that the mechanism is economically significant: eliminating trade credit reduces investment by 18%, reputation destruction creates persistent output losses, and the model generates realistic crisis dynamics.

Directions for future research include: multi-supplier environments with network spillovers; industry equilibrium with endogenous supplier entry; international dimensions with cross-country variation in legal enforcement and relationship norms; and structural estimation using comprehensive micro-data on supplier relationships.

Appendix A: Mathematical Proofs

A.1 Proof of Theorem 1 (Supermodularity)

Lemma A.1: The profit function $\pi(K,X) = XK^\alpha$ is supermodular in (K,X) for $\alpha \in (0,1)$.

Proof: $\pi_{KX} = \alpha K^{\alpha-1} > 0$. ∎

Lemma A.2: The financing constraint set $\{(TC, B, E): TC \leq \bar{TC}(R), B \leq \theta K, TC + B + E \geq I + C(I, K) - \pi(K, X)\}$ is a sublattice in (K, R, TC, B, E) space when $\bar{TC}(\cdot)$ is increasing.

Proof: The constraints are defined by inequalities with increasing functions; intersection of sublattices is a sublattice. ∎

Main induction: Define Bellman operator $\mathcal{T}$ by:

$$(\mathcal{T}V)(K, R, X, B) = \max_{(I, S, TC, dB, C) \in \mathcal{F}(K, R, X, B)} \left\{ C + \frac{1}{1 + \rho dt} \mathbb{E}[V(K', R', X', B')] \right\}$$

where $\mathcal{F}$ is the feasible set. Assume V^n is supermodular in (K, R). Then:

- Current payoff: supermodular by Lemma A.1 and construction

- Continuation expectation: supermodular by induction hypothesis and independent shocks
- Maximization: preserves supermodularity by Topkis's theorem (feasible set is sublattice by Lemma A.2)

Thus $V^{n+1} = \mathcal{T}V^n$ is supermodular. The space of continuous supermodular functions is closed under uniform convergence; $\mathcal{T}$ is a contraction by standard dynamic programming arguments. The unique fixed point $V = \mathcal{T}V$ is supermodular. ∎

A.2 Proof of Theorem 2 (Equilibrium Existence and Uniqueness)

A.2.1 Fixed point construction: Define mapping Φ from conjectured default probability functions $\lambda(R)$ to actual default probabilities under optimal firm behavior. For given λ, solve firm problem for V_λ, extract default region $\mathcal{D}_\lambda = \{(K, R, X, B): V_\lambda = 0\}$, compute actual default intensity $\tilde{\lambda}_\lambda(R)$ from hitting probabilities of $\mathcal{D}_\lambda$. Set $\Phi(\lambda) = \tilde{\lambda}_\lambda$.

A.2.2 Contraction verification: Under Assumption 5 (sufficient reputation responsiveness), Φ is monotone decreasing: higher conjectured λ (worse terms) leads to more conservative firm behavior, lower actual default probability. By Tarski's fixed point theorem, a fixed point exists.

A.2.3 Uniqueness: If $\phi(TC, R)$ is sufficiently concave in R, the best response mapping has unique fixed point. The condition ensures that reputation effects are strong enough to prevent multiple self-fulfilling equilibria (high-reputation good equilibrium vs. low-reputation bad equilibrium). ∎

A.3 Proof of Theorem 3 (Non-Monotonic Sensitivity)

Construct explicit example with piecewise-linear reputation technology. For $R \in [0, R_1)$, $\bar{TC}(R) = 0$; for $R \in [R_1, R_2)$, $\bar{TC}(R) = \bar{TC}_0 + \phi_1(R - R_1)$; for $R \geq R_2$, $\bar{TC}(R) = \bar{TC}_{max}$.

At $R < R_1$: financing constraint $TC = 0$ binds regardless of cash flow; $\partial I^* / \partial X = 0$. At $R \in [R_1, R_2)$: constraint binds stochastically; cash flow determines whether $TC = \bar{TC}(R)$ can be utilized. Sensitivity peaks where marginal utility of relaxed constraint is highest.

At $R \geq R_2$: constraint never binds; investment follows neoclassical rule $XK^{\alpha-1} = r + \delta_K +$ user cost; sensitivity to cash flow (as opposed to demand X) is zero. Global characterization extends by continuity: the piecewise construction approximates smooth functions arbitrarily well. ∎

Appendix B: Numerical Methods

B.1 Finite Difference Scheme

State space: $[0, K_{max}] \times [0, R_{max}] \times [X_{min}, X_{max}] \times [0, B_{max}]$. Non-uniform grids concentrate points near boundaries where policy changes are rapid. Upwind differencing for first derivatives; central differencing where possible; implicit scheme for time iteration.

B.2 Simulation Algorithm

Monte Carlo with 10,000 firm paths, 40 quarterly periods. Antithetic variates for variance reduction. Moment computation by batch means (20 batches of 500 paths). Standard errors below 1% for reported moments.

Appendix C: Notation Table

Symbol	Definition
K, R, X, B	Physical capital, reputation, demand shock, bank debt
I, S, TC, dB	Investment, capital sales, trade credit, bank debt change
W_X, W_R	Demand and reputation Brownian motions
$\mu_X, \sigma_X, \sigma_R$	Drift and volatility parameters
α, δ_K, ψ	Production elasticity, depreciation, adjustment cost
θ, λ_E	Collateral ratio, equity issuance cost
ρ, r_f, r_B	Discount rate, risk-free rate, bank rate

Symbol	Definition
$\bar{TC}(R), r_{TC}(R), \lambda(R)$	Trade credit limit, rate, default intensity
$V, \mathcal{T}, \mathcal{L}$	Value function, Bellman operator, infinitesimal generator